1 General questions of growth:

- What are the determinants of long-run economic growth?
- How can we explain the vast differences in both output levels and growth rates across countries/time? For example, the average worker in the richest country earns about 35 times the income of an average worker in the poorest country.
- Small differences in growth rates make huge differences in per-capita output levels over time:
  - The real per-capita GDP in the U.S. grew by a factor of 8.1 from $2244 in 1870 to $18,258 in 1990. A 1.75% per-year growth rate.
  - Pakistan grew 0.88% per-year; India 0.64%, Philippines 0.86%.
  - Counterfactual: If U.S. had grown at 0.75% instead: U.S. per-capita GDP in 1990 would have been $5519. (would have ranked 37th in the world; instead of 1st)
- There is high mobility of countries in the world income distribution. About 60% of countries in the middle of the distribution moved either up or down the world income distribution between 1960 and 1985.[1]
- There has been both miracle and disaster countries.

2 Neoclassical growth theory

- Classical economists, such as Adam Smith (1776), David Ricardo (1817), Thomas Malthus (1798), and later Frank Ramsey (1928), Allyn Young (1928), Frank Knight (1944) Joseph Schumpeter (1934) provide the building blocks for the modern growth theory.
  - basic approaches to competitive behavior
  - equilibrium dynamics
  - role of diminishing returns
  - effects of technological progress, etc.
- Starting point: Ramsey (1928): Use of household optimization, utility theory : used in asset prices, growth, consumption behavior etc.
- Solow (1956) and Swan (1956) Introduction of the neoclassical production function: CRS, diminishing returns to each input. Assumed constant saving rate, very simple general equilibrium model. One problem, technological progress was not modelled, so diminishing returns eventually resulted in a state with no more growth.

[1] Chari, Kehoe, and McGrattan (1977)

- 1950s and 60s, exogenous technical progress is added.
- Cass (1965) and Koopmans (1965) provided a framework with endogenous savings. Framework became very technical. Technological progress still exogenous.
- Endogenous growth : Romer (1986), Lucas (1988), Rebelo (1991).

2.1 Solow Model

Solow's specific question: What do simple neoclassical assumptions imply about growth? His key assumptions include:

- Constant returns to scale.
- Perfect competition.
- Complete information.
- No externalities.

2.1.1 The basic model

Time is discrete. The economy has one consumer with infinite lifetime, and one firm.

Consumer

The consumer supplies labor L_t to the market, at market wage w_t. The consumer also owns all of the capital K_t and rents to the market at rental rate r_t. The consumer also owns the firm and receives its total profit π_t. Income is thus:

$$Y_t = r_t K_t + w_t L_t + \pi_t.$$

Labor supply grows at exogenous rate n: $L_t = (1+n)L_t$.

Even though there is only one consumer, the fact that his labor supply is growing over time captures the idea of an expanding population. Capital is accumulated by the consumer and depreciates at rate δ : $K_{t+1} = (1-\delta)K_t + I_t$. where I_t is gross investment. Notice that the price of capital is one.

Solow assumed:

1. The consumer saves some exogenous fraction s of income.

2. The consumer supplies all of his labor and capital inelastically. So $K_{t+1} = (1-\delta)K_t + sY_t$

Firm

The firm can take capital and labor and convert it into output (consumption and new capital) which is then sold back to the consumer. The firm's technology is described by the production function $Y_t = F(K_t, A_t L_t)$. A_t is the level of "technology" at time t. It grows at exogenous rate g: $A_{t+1} = (1+g)A_t$

This is labor augmenting technological progress, meaning that it increases the effective amount of labor.

The neoclassical production function $F(K, L)$ has the following properties:

1. F is homogeneous of degree 1. Formally, for any $\lambda \geq 0$, $F(\lambda K, \lambda L) = \lambda F(K, L)$. In other words, production exhibits constant returns to scale.

2. Both factors are necessary, i.e.,$F(K, 0) =F(0, L) = 0$, for any K, L.

3. Both factors contribute to output:

4. $\frac{\vartheta F(K,L)}{\vartheta K} > 0$ and $\frac{\vartheta F(K,L)}{\vartheta L} > 0$

5. The firm has decreasing returns in each product, or F is concave in both arguments:
   $\frac{\vartheta^2 F(K,L)}{\vartheta K^2} < 0$ and $\frac{\vartheta^2 F(K,L)}{\vartheta L^2} < 0$

6. Inada conditions hold
   $\lim_{K\to 0} \frac{\vartheta F(K,L)}{\vartheta K} = \infty$ and $\lim_{K\to\infty} \frac{\vartheta F(K,L)}{\vartheta K} = 0$

7. The firm's profits are $\pi_t = F(K_t, A_t L_t) - w_t L_t - r_t K_t$. Because firm profits go to the consumer, the consumer's income is equal to the firm's total output.

Markets

All goods are traded on a competitive market. Whenever we write down a macroeconomic model, we usually have to define an "equilibrium" in the model is.

An equilibrium for this model is a sequence of factor prices $\{w_t\}$, $\{r_t\}$, and allocations $\{K_t, L_t\}$ such that

1. The capital stock and labor supply and technology level are determined by equations $K_{t+1} = (1 - \delta)K_t + I_t$, $L_t = (1 + n)L_t$, $A_{t+1} = (1 + g)A_t$ and initial conditions K_0, L_0 and A_0, respectively.

2. Taking prices as given, the firm purchases capital K_t and labor L_t to maximize its profits.

3. Markets clear, that is, the capital and labor demand of the firm at prices w_t and r_t are equal to the supply.

The main methodological failing of the model from a current perspective is that the actions of the consumer are simply assumed. In a modern macro model you are expected to write down a utility function and set of budget constraints for the representative consumer, and derive the consumer's optimal actions. Notice that this model will be useless as a means of analyzing business cycles (we need shocks to AD or AS), monetary policy (we need money); unemployment (we need either a labor/leisure trade-off, or some reason why labor markets don't clear); welfare effects of policy (we need a utility function and policy instruments). However, it will prove very useful for analyzing the relative contribution of capital and technical progress to economic growth.

2.1.2 Dynamics of the Solow Model

We have a system of difference equations

$K_{t+1} = (1-\delta)K_t + sY_t$ where $Y_t = F(K_t, A_tL_t)$.

$L_t = (1+n)L_t$

$A_{t+1} = (1+g)A_t$ and initial conditions K_0, L_0, A_0. To solve this first define:

$$k_t = \frac{K_t}{A_tL_t}. \text{ and } y_t = \frac{Y_t}{A_tL_t}$$

where k_t and y_t are output per worker and capital per worker. With growing productivity, they are: capital and output per "effective worker".

$$\frac{K_{t+1}}{A_tL_t} = (1-\delta)\frac{K_t}{A_tL_t} + s\frac{F(K_t, A_tL_t)}{A_tL_t}$$

Since F is homogeneous of degree 1

$$\frac{F(K, AL)}{AL} = F(\frac{K}{AL}, 1).$$

So we get

$$\frac{K_{t+1}}{A_tL_t} = (1-\delta)\frac{K_t}{A_tL_t} + sF(\frac{K_t}{A_tL_t}, 1)$$

Multiply and divide the LHS with $A_{t+1}L_{t+1}$ and substitute the per capita definitions to get:

$$k_{t+1} = \frac{(1-\delta)k_t + sF(k_t, 1)}{(1+g)(1+n)}.$$

Define a new function:

$$f(k) = F(k, 1)$$

so that

$$y_t = f(k_t)$$

where f has the same properties as F but is not homogeneous of degree 1.

$$k_{t+1} = \frac{(1-\delta)k_t + sf(k_t)}{(1+g)(1+n)}$$

This is a first-order difference equation describing the evolution of "capital per effective worker" over time. We can write it as: $k_{t+1} = d(k_t)$. What are the properties of d where

$$d(k_t) = \frac{(1-\delta)k_t + sf(k_t)}{(1+g)(1+n)}.$$

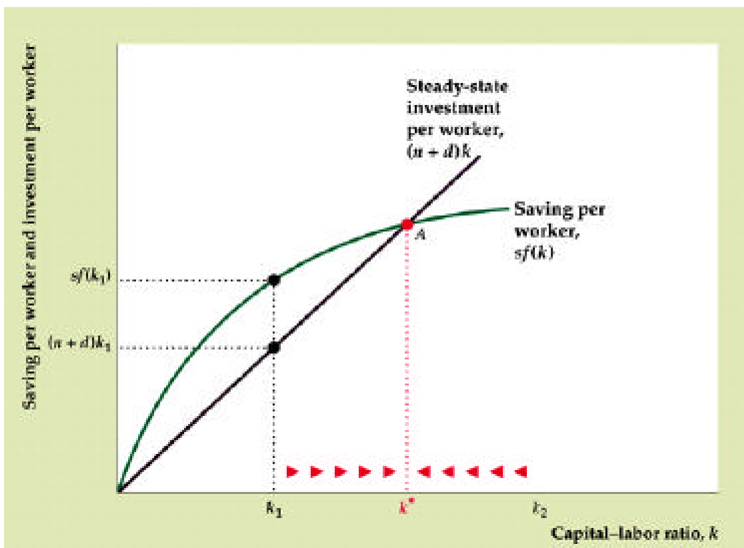
- If $k_t = 0$, $d(k_t)$ is zero. So, it passes through the origin.
- $d'(k) > 0$; strictly increasing.
- $d''(k) < 0$; strictly concave.

- $\lim_{k\to 0} d'(k) = \infty$ and $\lim_{k\to\infty} d'(k) = \frac{(1-\delta)}{(1+g)(1+n)} \in [0, 1]$

  Steady states is a fixed point where once the economy get there it stays there forever. Finding steady states involves solving $k_\infty = d(k_\infty)$. Initially assume that $g = 0$.

  In the following graph steady state is at a point where total savings equals total depreciation. (n is the population growth)

Figure 6.03 Determining the capital-labor ratio in the steady state



From any initial positive level of the capital stock, k_t will converge monotonically to a steady state k^*. What does this imply about the aggregate variables we are interested in? If we are at the steady state, then:

- Since $k_t = \frac{K_t}{A_t L_t}$ at the steady state, K_t and $A_t L_t$ will be growing at the same rate.
- Since the growth rate of A_t is g and the growth rate of L_t is n, the growth rate of K_t is $g + n$
- Since F is homogeneous of degree 1, Y_t is also growing at rate $g + n$
- Per capita output grows at rate g.

The steady state for k_t corresponds to a "balanced growth path" for the original variables. A balanced growth path is a sequence in which all of the variables grow at a constant rate. Notice that savings rates do not enter into long run growth rates. The only thing that matters for long-run growth in per capita output is the growth rate of technology. Increased savings only affects the level of per capita output.

Example 1: Suppose $g = n = 0$, $Y_t = Ak_t^{\alpha}$. Notice that in this example there is no growth at the steady state.

Steady state capital stock will be obtained from:

$$k^* = (1-\delta)k^* + sAk^{*\alpha}$$

which implies:

$$\delta k^* = sAk^{*\alpha}.$$

This equation specifies that at the steady state the amount that is saved will equal total depreciation. Consequently, capital stock will not grow any more. Notice that if there is population growth, we would modify this equation as:

$$(n+\delta)k^* = sAk^{*\alpha}$$

In the case without population growth we will get:

$$k^* = (\frac{sA}{\delta})^{\frac{1}{1-\alpha}}$$

and $y^* = f(k^*)$; $c_s = (1-s)y_s$ and investment $x_s = sy^*$.

- In this model, investment rate and capital to output ratios are given by:

$$\frac{x_t}{y_t} = s \text{ and } \frac{k_t}{y_t} = \frac{k_t^{1-\alpha}}{A}$$

which implies that at the steady state

$$\frac{k^*}{y^*} = \frac{s}{\delta}.$$

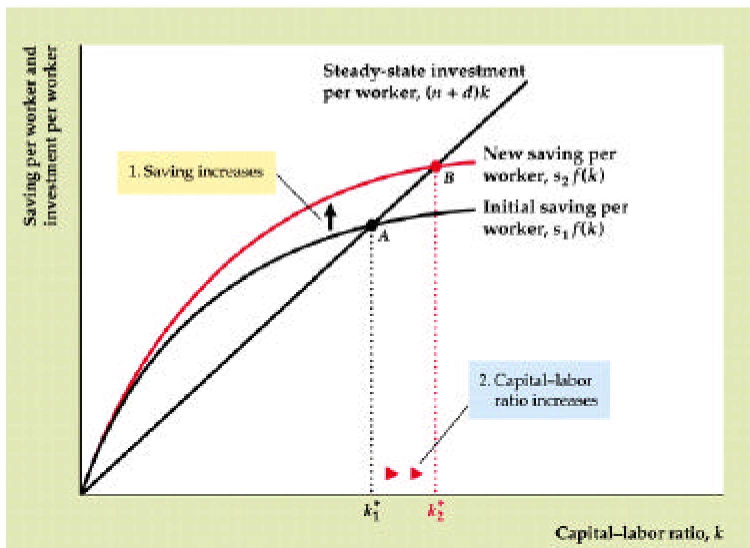
Notice that capital output ratio does not depend on A. What are the implications of this model on output differences across countries? Assume that countries are identical except for the differences in saving rates.

- In the steady state of this example:

$$\frac{(x^*/y^*)^i}{(x^*/y^*)^j} = \frac{(k^*/y^*)^i}{(k^*/y^*)^j} = \frac{s^i}{s^j}, \text{ and } \frac{y^i}{y^j} = \left(\frac{s^i}{s^j}\right)^{\frac{\alpha}{1-\alpha}}$$

- Therefore Solow model implies that countries that save more will have higher investment rates and higher capital output ratios. Per-worker income levels will be higher in countries that save more. The graph below which is taken from Abel and Bernanke shows this result. Notice that the notation is slightly different

Figure 6.04 The effect of an increased saving rate on the steady-state capital-labor ratio



Can this model account for observed differences in per-worker income levels across countries? Lets consider some reasonable set of parameters: A=1 (normalization); $\alpha = 1/3$ (capital income share), $s^i = 0.3$ (investment rate in the U.S.), $\delta = .10$. Now consider another economy with a different saving rate. What kind of differences in per-capita income levels can we generate with differences in saving rates?

Solow Model

$\frac{s^i}{s^j}$	$\frac{y^i}{y^j}$	$\frac{y^i}{y^j}$ with $a = 2/3$
2	1.4	4
5	2.2	25
10	3.2	100

Second column of the table shows very small differences in output per-capita across countries. In the data the differences in per-capita output levels are

Figure 6.05 The effect of a higher population growth rate on the steady-state capital-labor ratio

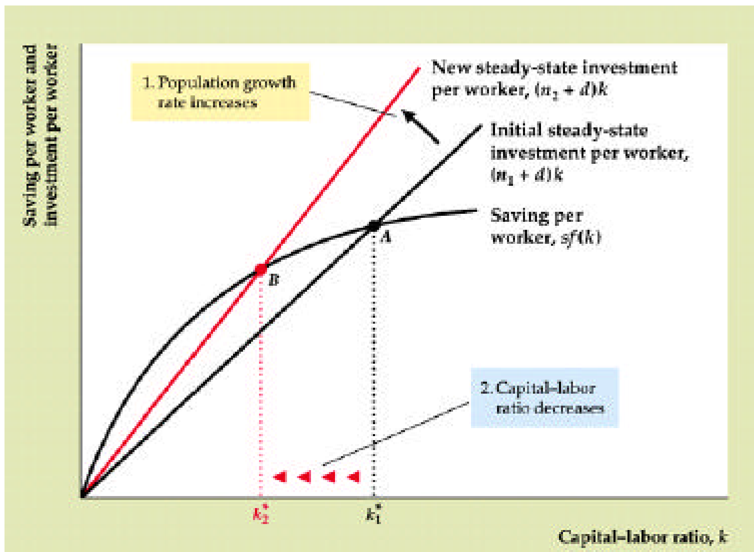


Figure 1:

around 30-35. This conclusion is very sensitive to the value of the capita share. If $a = 2/3$. Sometimes a broader definition of capital can be used which leads to higher share of capital. However, this approach is not exactly satisfactory in explaining differences in incomes per capita across countries.

- **Example 2:** Now assume that countries differ in their levels of TFP as well.
  - The Solow model implies that:

$$\frac{(x^*/y^*)^i}{(x^*/y^*)^j} = \frac{(k^*/y^*)^i}{(k^*/y^*)^j} = \frac{s^i}{s^j}, \text{ and } \frac{y^i}{y^j} = \left(\frac{A^i}{A^j}\right)^{\frac{1}{1-\alpha}} \left(\frac{s^i}{s^j}\right)^{\frac{\alpha}{1-\alpha}}$$

  - In this case, investment rates and capital output ratio are not affected by TFP differences across countries. However, relative income differences depend on both differences in TFP and in saving rates.
  - Issues here involve how to identify differences in TFP levels across countries and to find out determinants of TFP.

2.1.3 Implications of the Solow model on Convergence.

- How does the initial level of capital affect growth rates?
- Convergence - poor countries grow faster than rich countries.
- Divergence - rich countries grow faster than poor countries.
- Assume that all countries have the same savings rates and access to the same technology. Then the model implies that all countries are moving towards the same long-run income level. (absolute convergence)
- Assume that all countries have the same savings rates and rates of technological advance (though maybe different initial levels of technology) This implies that poor countries grow faster than rich countries.
- Countries may have different savings rates, and as a result be moving towards different long-run income levels. However, if you control for the various determinants of long-run income (in particular savings rates), initial income will have a negative correlation with growth rates. (Conditional convergence)

  There's a huge empirical literature on convergence. Stylized facts:

- Absolute convergence among U.S. states, Canadian provinces, Japanese prefectures, European regions, etc. (Barro)
- Absolute convergence among "open" economies (Sachs and Warner1995).
- No evidence for absolute convergence among the countries of the world.(Romer 1986)
- Strong evidence for conditional convergence. (Mankiw, Romer and Weil 1992, Barro (many papers), many others)
- Consensus developing: suitably similar economies, with economic ties between them, have a strong tendency to converge. For example, one would expect to find income gaps between EC countries to narrow. Countries with vast differences in institutions, infrastructure, etc., may not converge.
- Solow model has implications on the speed of convergence as well. Barro(1995). For typical parameters the speed of convergence implied by the model is too fast.

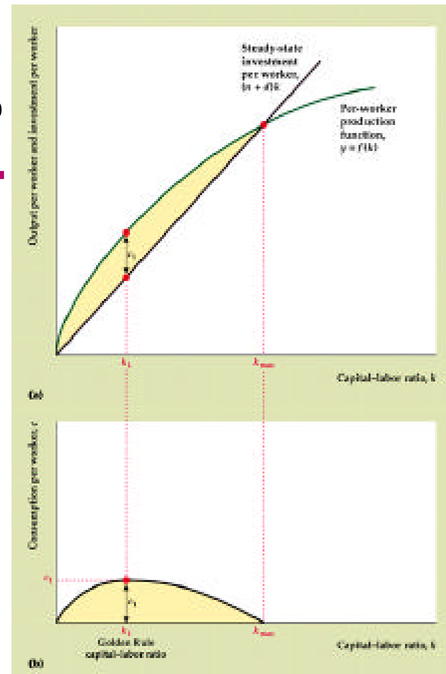
2.1.4 Golden Rule.

For a given production function and given values of the growth rates and the depreciation rate, there is a unique steady state value of the capital stock for each value of the saving rate. Denote this relation as $k^*(s)$. The steady state level of per capita consumption is $c^* = (1-s)f(k^*(s)) = f(k^*(s) - (n+\delta)k^*(s)$

where n is the growth rate of population. Next figure shows the relationship between saving rates and consumption.

Figure 6.2

The relationship of consumption per worker to the capital-labor ratio in the steady state



The golden rule saving rate is the one where $(n + \delta) = f'(k)$

2.2 Growth Accounting

How much of a country's growth can be explained by:

- Labor force growth
- Capital accumulation
- Technical progress

Solow's model allows us to decompose growth into these three components. Assume a Cobb-Douglas production function:

$$Y_t = A k_t^{\alpha} L_t^{1-a}$$

If we consider a situation in which the capital stock, the labor force, and total factor productivity are changing, then the proportional growth rate of output is:

$$\frac{\Delta Y}{Y_t} = a\frac{\Delta K}{K_t} + (1-a)\frac{\Delta L}{L_t} + \frac{\Delta A}{A_t}$$

with $a\frac{\Delta K}{K_t}$ giving the contribution of capital to the growth of output, $(1-a)\frac{\Delta L}{L_t}$ giving the contribution of labor to the growth of output, and $\frac{\Delta A}{A_t}$ giving the contribution of total factor productivity to the growth of output. If we know the proportional growth rates of output, the capital stock, and the labor force, and if we know the capital share parameter in the production function, then we can use this growth-accounting equation to calculate the (not directly observed) rate of growth of total factor productivity A, and to decompose the growth of total output Y into (i) the contribution from the increasing capital stock K, (ii) the contribution from the increasing labor force L, and (iii) the contribution from higher total factor productivity A. Last term is often refereed to as the Solow Residual. So what is the Solow residual?

1 - the part of growth which is not explained by capital accumulation and labor force expansion. "A measure of our ignorance," as Solow himself put it.

2- the part of growth which is explained by technical progress.

Example 1: These are from a famous paper by Alwyn Young (1994 QJE) called "The tyranny of numbers: confronting the statistical realities of the East Asian growth experience".

Country	α	**GDP gr**	**From K**	**From L**	**From A**	**% of gr dueTFP**
Hong Kong	37	7.3	3.09	2.00	2.20	30.1
Singapore	53 8	8.50	6.20	2.86	-0.40	
South Korea	32	10.32	4.77	4.35	1.20	11.6
Taiwan	29	9.10	3.68	3.62	1.80	19.8

The results in this table ran counter to the very popular idea at the time that these countries experienced a "miracle" increase in productivity. Instead, the table points out that the high growth rates in these four countries can be traced primarily to rapid capital accumulation and labor force growth.

This result has been challenged by K-R.

Example 2:

Growth Accounting in United States				
(percent per year)				
	1929-1948	1948-1973	1973-1982	1929-1982
Source of growth				
labor growth	1.42	1.40	1.13	1.34
capital growth	0.11	0.77	0.69	0.56
total input growth	1.53	2.17	1.82	1.90
productivity growth	1.01	1.53	−0.27	1.02
total output growth	2.54	3.70	1.55	2.92