

# Modeling Default Risk

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## Overview

Default risk is the uncertainty surrounding a firm's ability to service its debts and obligations. Prior to default, there is no way to discriminate unambiguously between firms that will default and those that will not. At best we can only make probabilistic assessments of the likelihood of default. As a result, firms generally pay a spread over the default-free rate of interest that is proportional to their default probability to compensate lenders for this uncertainty.

Default is a deceptively rare event. The typical firm has a default probability of around 2% in any year. However, there is considerable variation in default probabilities across firms. For example, the odds of a firm with a AAA rating defaulting are only about 2 in 10,000 per annum. A single A-rated firm has odds of around 10 in 10,000 per annum, five times higher than a AAA. At the bottom of the rating scale, a CCC-rated firm's odds of defaulting are 4 in 100 (4%), 200 times the odds of a AAA-rated firm.

The loss suffered by a lender or counterparty in the event of default is usually significant and is determined largely by the details of the particular contract or obligation. For example, typical loss rates in the event of default for senior secured bonds, subordinated bonds and zero coupon bonds are 49%, 68%, and 81%, respectively.

Cross default clauses in debt contracts usually ensure that the default probabilities for each of the classes of debt for a firm are the same. That is, the default probability of the firm determines the default probability for all of the firm's debt or counterparty obligations. However, the loss in the event of default for each of the classes of obligations can vary widely depending on their nature (security, collateral, seniority, etc.).

Although in general a poor investment strategy, it is possible to be rewarded for taking on large concentrations of risk in equities because these concentrations at times produce large returns. However, overwhelming evidence of the ineffectiveness of this *stock-picking* strategy has been available since the early 1970s and, as a result, the majority of equity investments are managed in diversified portfolios. Unlike equities, debt has no upside potential and thus the case for managing default risk in well-diversified portfolios is even more compelling. The limited upside potential of debt spreads means that there are no possible circumstances under which an investor or counterparty can be rewarded for taking on concentrations of default risk. Like other rare events with high costs, default risk can only be effectively managed in a portfolio.

In addition to knowing the default probability and loss given default, the portfolio management of default risk requires the measurement of default correlations. Correlations measure the degree to which the default risks of the various borrowers and counterparties in the portfolio are related. The elements of credit risk can therefore be grouped as follows:

### Standalone Risk

*Default probability*, the probability that the counterparty or borrower will fail to service its obligations.

*Loss given default*, the extent of the loss incurred in the event the borrower or counterparty defaults.

*Migration risk*, the probability and value impact of changes in default probability.

### Portfolio Risk

*Default correlations*, the degree to which the default risks of the borrowers and counterparties in the portfolio are related.

*Exposure*, the size, or proportion, of the portfolio exposed to the default risk of each counterparty and borrower.

While each of these items is critical to the management of credit portfolios, none are more important or more difficult to determine, than the default probability. The remainder of this paper will focus on the determination of default probability using information from a firm's financial statements and the market price of its equity.

## Measuring Default Probability: The Problem

There are three main elements that determine the default probability of a firm:

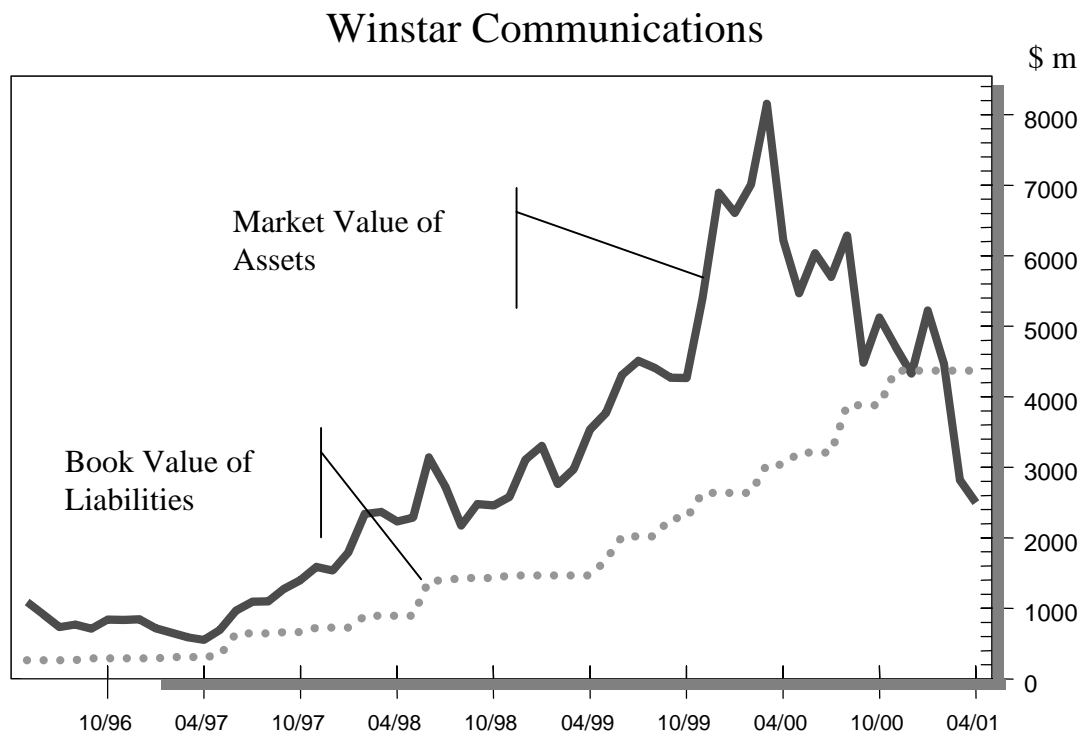
**Value of Assets:** the *market value* of the firm's assets. This is a measure of the present value of the future free cash flows produced by the firm's assets discounted back at the appropriate discount rate. This measures the firm's prospects and incorporates relevant information about the firm's industry and the economy.

**Asset Risk:** the *uncertainty or risk* of the asset value. This is a measure of the firm's business and industry risk. The value of the firm's assets is an estimate and is thus uncertain. As a result, the value of the firm's assets should always be understood in the context of the firm's business or asset risk.

**Leverage:** the extent of the firm's contractual liabilities. Whereas the relevant measure of the firm's assets is always their market value, the book value of liabilities relative to the market value of assets is the pertinent measure of the firm's leverage, since that is the amount the firm must repay.

For example, Figure 1 illustrates the evolution of the asset value and book liabilities of Winstar Communications, a New York Telephone company that filed for Chapter 11 bankruptcy protection in April 2001.

The default risk of the firm increases as the value of the assets approaches the book value of the liabilities, until finally the firm defaults when the market value of the assets is insufficient to repay the liabilities.



**Figure 1**

In our study of defaults, we have found that in general firms do not default when their asset value reaches the book value of their total liabilities. While some firms certainly default at this point, many continue to trade and service their debts. The long-term nature of some of their liabilities provides these firms with some breathing space. We have found that the *default point*, the asset value at which the firm will default, generally lies somewhere between total liabilities and current, or short-term, liabilities.

The relevant net worth of the firm is therefore the market value of the firm's assets minus the firm's default point:

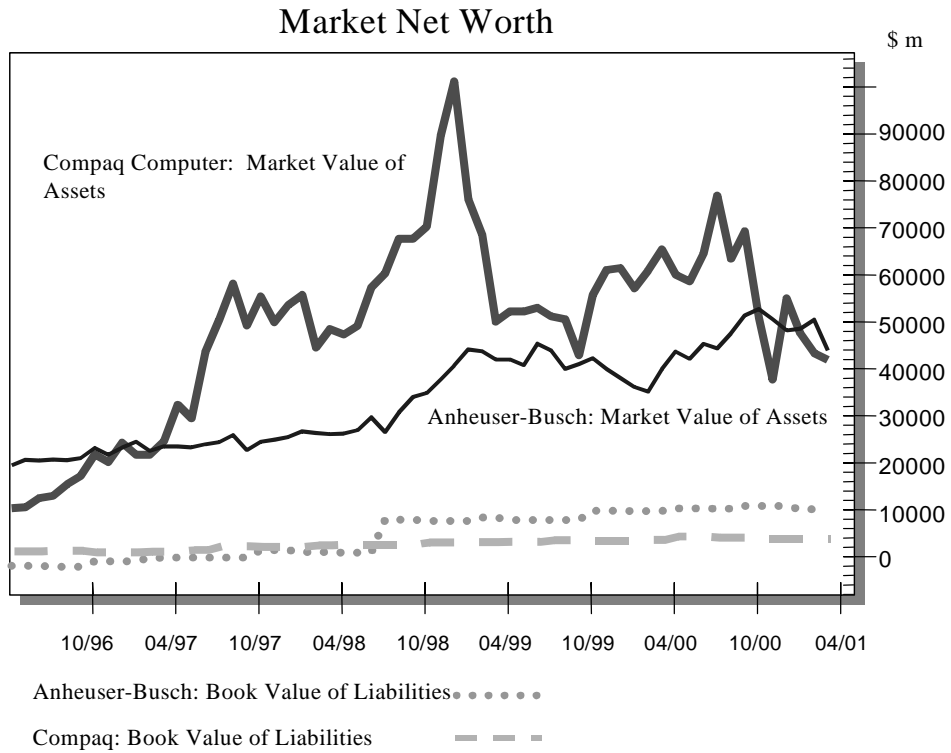
$$\left[ \begin{array}{c} \text{Market Value} \\ \text{of Assets} \end{array} \right] - \left[ \begin{array}{c} \text{Default} \\ \text{Point} \end{array} \right]$$

A firm will default when its market net worth reaches zero.

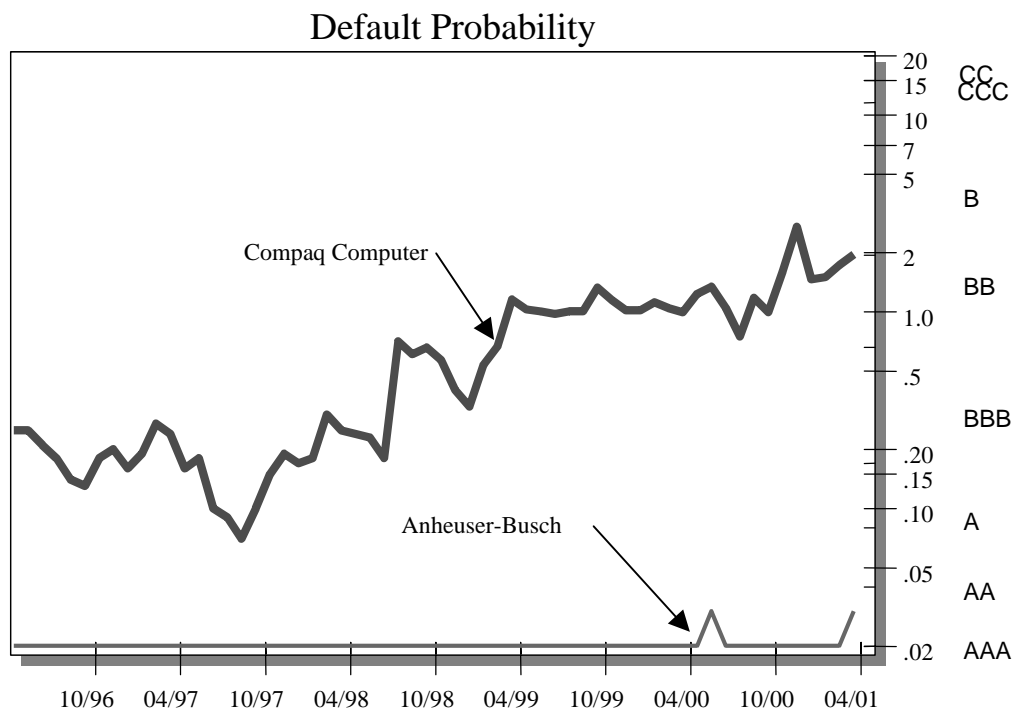
Like the firm's asset value, the market measure of net worth must be considered in the context of the firm's business risk. For example, firms in the food and beverage industries can afford higher levels of leverage (lower market net worth) than high technology businesses because their businesses, and consequently their asset values, are more stable and less uncertain.

For example, Figure 2 shows the evolution of asset values and default points for Compaq Computer and Anheuser-Busch. Figure 3 shows the corresponding evolution of the annual

default probabilities. The default probabilities shown in this figure are the one year default rates, the probability that the firm will default in the ensuing year, and are displayed on a logarithmic scale.



**Figure 2**



**Figure 3**

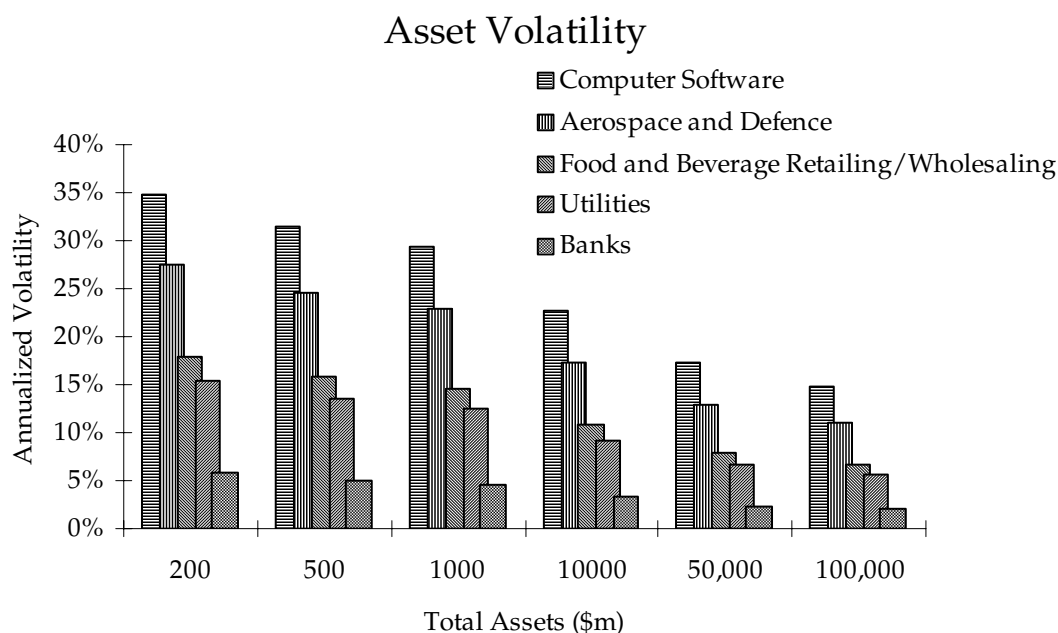
The effect of the relative business risks of the two firms is clear from a comparison of the two figures. For instance, as of April 2001, the relative market values, default points, asset risks and resulting default probabilities for Compaq and Anheuser-Busch were:

	Anheuser-Busch	Compaq Computer
Market Value of Assets	44.1	42.3
Default Point	5.3	12.2
Market Net Worth (\$b)	38.8	30.1
Asset Volatility	21%	39%
Default Probability (per annum)	.03%	1.97%

The asset risk is measured by the asset volatility, the standard deviation of the annual percentage change in the asset value. For example, Anheuser-Busch's business risk is 21%, which means that a one standard deviation move in their asset value will add (or remove) \$9 bn from its asset value of \$44.1 bn. In contrast, a one standard deviation move in the asset value of Compaq Computer will add or remove \$16.5 bn from its asset value of \$42.3 bn. The difference in their default probabilities is thus driven by the difference in the risks of their businesses, not their respective asset values or leverages.



As you would expect, asset volatility is related to the size and nature of the firm's business. For example, Figure 4 shows the asset volatility for several industries and asset sizes.



**Figure 4**

Asset volatility is related to, but different from, equity volatility. A firm's leverage has the effect of magnifying its underlying asset volatility. As a result, industries with low asset volatility (for example, banking) tend to take on larger amounts of leverage while industries with high asset volatility (for example, computer software) tend to take on less. As a consequence of these compensatory differences in leverage, equity volatility is far less differentiated by industry and asset size than is asset volatility.

Asset value, business risk and leverage can be combined into a single measure of default risk which compares the market net worth to the size of a one standard deviation move in the asset value. We refer to this ratio as the *distance-to-default* and it is calculated as:

$$\left[ \begin{array}{l} \text{Distance} \\ \text{to Default} \end{array} \right] = \frac{\left[ \begin{array}{l} \text{Market Value} \\ \text{of Assets} \end{array} \right] - \left[ \begin{array}{l} \text{Default} \\ \text{Point} \end{array} \right]}{\left[ \begin{array}{l} \text{Market Value} \\ \text{of Assets} \end{array} \right] \left[ \begin{array}{l} \text{Asset} \\ \text{Volatility} \end{array} \right]}.$$

For example, in April 2001 Anheuser-Busch was approximately 4.2 standard deviations away from default while, in contrast, Compaq Computer was only 1.8 standard deviations away from default. That is, it would take a 4.2 standard deviation move in the asset value of Anheuser-

Busch before it will default while only a 1.8 standard deviation move is required in Compaq's asset value to result in its default.

The distance-to-default measure combines three key credit issues: the value of the firm's assets, its business and industry risk, and its leverage. Moreover, the distance-to-default also incorporates, via the asset value and volatility, the effects of industry, geography and firm size.

The default probability can be computed directly from the distance-to-default if the probability distribution of the assets is known, or, equivalently, if the default rate for a given level of distance-to-default is known.

## Measuring Default Probability: A Practical Approach

There are three basic types of information available that are relevant to the default probability of a firm: financial statements, market prices of the firm's debt and equity, and subjective appraisals of the firm's prospects and risk. Financial statements, by their nature, are inherently backward looking. They are reports of the past. Prices, by their nature, are inherently forward looking. Investors form debt and equity prices as they anticipate the firm's future. In determining the market prices, investors use, amongst many other things, subjective appraisals of the firm's prospects and risk, financial statements, and other market prices. This information is combined using their own analysis and synthesis and results in their willingness to buy and sell the debt and equity securities of the firm. Market prices are the result of the combined willingness of many investors to buy and sell and thus prices embody the synthesized views and forecasts of many investors.

The most effective default measurement, therefore, derives from models that utilize both market prices and financial statements. There is no assertion here that markets are perfectly efficient in this synthesis. We assert only that, in general, it is difficult to do a better job than they are doing. That is, in general, it is very difficult to consistently beat the market. Consequently, where available, we want to utilize market prices in the determination of default risk because prices add considerably to the predictive power of the estimates.

Oldrich Vasicek and Stephen Kealhofer have extended the Black-Scholes-Merton framework to produce a model of default probability known as the Vasicek-Kealhofer (VK) model. This model assumes the firm's equity is a perpetual option with the default point acting as the absorbing barrier for the firm's asset value. When the asset value hits the default point, the firm is assumed to default. Multiple classes of liabilities are modeled: short-term liabilities, long-term liabilities, convertible debt, preferred equity, and common equity. When the firm's asset value becomes very large, the convertible securities are assumed to convert and dilute the existing equity. In addition, cash payouts such as dividends are explicitly used in the VK model. A default database is used to derive an empirical distribution relating the distance-to-default to a default probability. In this way, the relationship between asset value and liabilities can be captured without resorting to a substantially more complex model characterizing a firm's liability process.

KMV has implemented the VK model to calculate an Expected Default Frequency™ (EDF™) credit measure which is the probability of default during the forthcoming year, or years for firms with publicly traded equity (this model can also be modified to produce EDF values for firms without publicly traded equity.) The EDF value requires equity prices and certain items from financial statements as inputs. EDF credit measures can be viewed and analyzed within the context of a software product called Credit Monitor™ (CM). CM calculates EDF values for years 1 through 5 allowing the user to see a term structure of EDF values. KMV's EDF credit measure assumes that Default is defined as the nonpayment of any scheduled payment, interest or principal. The remainder of this section describes the procedure used by KMV to determine a public firm's probability of default.

There are essentially three steps in the determination of the default probability of a firm:

**Estimate asset value and volatility:** In this step the asset value and asset volatility of the firm is estimated from the market value and volatility of equity and the book value of liabilities.

**Calculate the distance-to-default:** The distance-to-default (DD) is calculated from the asset value and asset volatility (estimated in the first step) and the book value of liabilities.

**Calculate the default probability:** The default probability is determined directly from the distance-to-default and the default rate for given levels of distance-to-default.

### Estimate Asset Value and Volatility

If the market price of equity is available, the market value and volatility of assets can be determined directly using an options pricing based approach which recognizes equity as a call option on the underlying assets of the firm. For example, consider a simplified case where there is only one class of debt and one class of equity. See Figure 5.

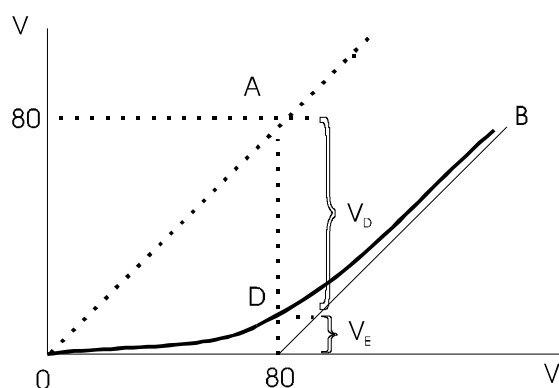
Assets	Liabilities
100	80
	20

**Figure 5**

The limited liability feature of equity means that the equity holders have the right, but not the obligation, to pay off the debt holders and take over the remaining assets of the firm. That is,

the holders of the other liabilities of the firm essentially own the firm until those liabilities are paid off in full by the equity holders. Thus, in the simplest case, equity is the same as a call option on the firm's assets with a strike price equal to the book value of the firm's liabilities.

The VK model uses this option nature of equity to derive the underlying asset value and asset volatility implied by the market value, volatility of equity, and the book value of liabilities. This process is similar in spirit to the procedure used by option traders in the determination of the implied volatility of an option from the observed option price.



**Figure 6**

For example, assume that the firm is actually a type of levered mutual fund or unit trust. The assets of the firm are equity securities and thus can be valued at any time by observing their market prices. Further, assume that our little firm is to be wound up after five years and that we can ignore the time value of money (discounting adds little to our understanding of the relationships and serves only to complicate the picture). That is, in five years time, the assets will be sold and the proceeds divided between the debt and equity holders.

Initially, assume that we are interested in determining the market value of the equity from the market value of the assets. This is the reverse of the problem we face in practice but provides a simpler perspective to initially understand the basic option relationships. See Figure 6.

To be specific, assume that we initially invest \$20 in the firm and borrow a further \$80 from a bank. The proceeds, \$100, are invested in equities. At the end of five years what is the value of equity? For example, if the market value of the assets at the end of year five is \$60 then the value of equity will be zero. If the value of the assets is \$110, then the value of the equity will be \$30, and so on. Thus, in Figure 6, the lines from \$0 to \$80 and from \$80 to point B represent the market value of the equity as a function of the asset value at the end of year five.

Assume now that we are interested in valuing our equity prior to the final winding up of the firm. For example, assume that three years have passed since the firm was started and that there are two years remaining before we wind the firm up. Further, we have marked the equities to market and their value is determined to be \$80. What is the value of the equity? Not zero. It is actually something greater than zero because it is the value of the assets two years hence that really matters and there is still a chance that the asset value will be greater than \$80

in two years time. In Figure 6, the value of the equity with two years to go is represented by the curve joining \$0 and point B.

The higher the volatility of the assets the greater is the chance of high asset values after two years. For example, if we were dissatisfied with our fund's performance after three years because it has lost \$20 in value, dropping from \$100 to \$80, we may be tempted to invest in higher-potential, higher-risk, equities. If we do, what is the effect on the equity value? It increases. The more volatile assets have higher probabilities of high values, and consequently, higher payouts for the equity. Of course there are accompanying higher probabilities of lower asset values, because volatility works both ways, but with limited liability this does not affect the equity value. At the end of the five years, it makes no difference to the equity if the final asset value is \$79 or \$9; its payout is the same, 0.

Where did the increase in the equity's value come from? It did not come from an increase in the asset value. We simply sold our original portfolio for \$80 and purchased a new portfolio of higher-risk equities for \$80. There was no value created there. The value of course came from the bank holding our firm's debt. In Figure 6, the value of the firm can be divided between the debt and equity holders along the line joining the points \$80 and A, where the line 0 to A plots the asset value against itself. Thus, the only way the value of equity can increase while the asset value remains constant is to take the value from the market value of the debt. This should make sense. When we reinvested the firm's assets in higher-risk equities, we increased the default risk of the debt and consequently reduced its market value.

The value of debt and equity are thus intimately entwined. They are both really derivative securities on the underlying assets of the firm. We can exploit the option nature of equity to relate the market value of equity and the book value of debt to determine the implied market value of the underlying assets. That is, we solve the reverse of the problem described in our simple example. We observe the market value of the equity and solve backwards for the market value of assets, see Figure 7.

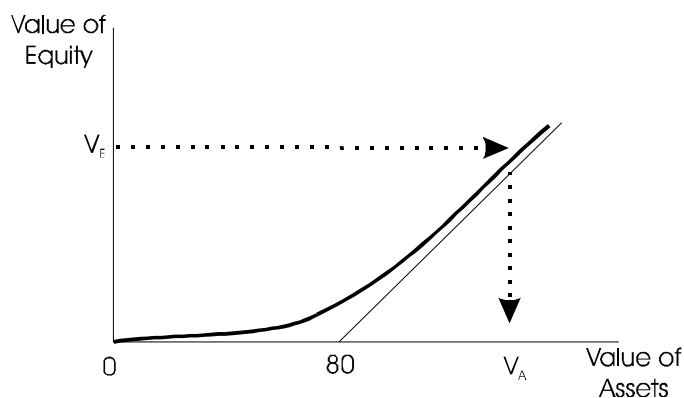


Figure 7

In practice, we need to take account of the more complex capital structures and situations that exist in real life. For example we need to consider the various terms and nature of debt (for example, long- and short-term debt, and convertible instruments), the perpetuity nature of

equity, the time value of money, and of course we also have to solve for the volatility of the assets at the same time. Thus, in practice, we solve the following two relationships simultaneously:

$$\begin{bmatrix} \text{Equity} \\ \text{Value} \end{bmatrix} = \text{OptionFunction} \left( \begin{bmatrix} \text{Asset} \\ \text{Value} \end{bmatrix}, \begin{bmatrix} \text{Asset} \\ \text{Volatility} \end{bmatrix}, \begin{bmatrix} \text{Capital} \\ \text{Structure} \end{bmatrix}, \begin{bmatrix} \text{Interest} \\ \text{Rate} \end{bmatrix} \right)$$

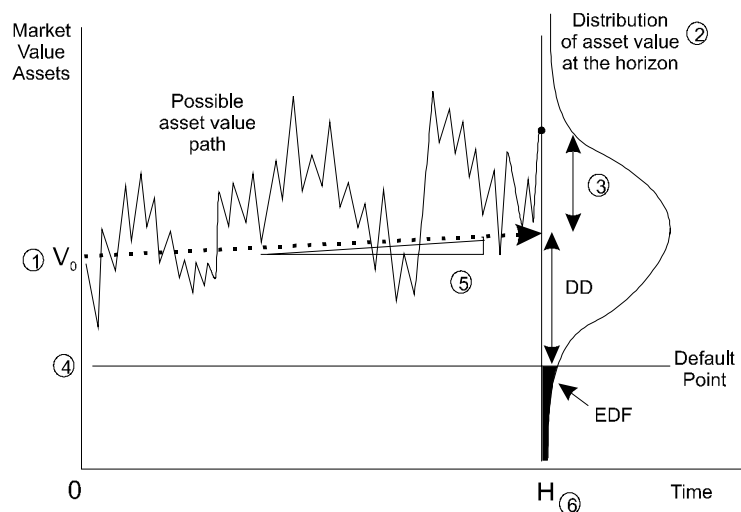
$$\begin{bmatrix} \text{Equity} \\ \text{Volatility} \end{bmatrix} = \text{OptionFunction} \left( \begin{bmatrix} \text{Asset} \\ \text{Value} \end{bmatrix}, \begin{bmatrix} \text{Asset} \\ \text{Volatility} \end{bmatrix}, \begin{bmatrix} \text{Capital} \\ \text{Structure} \end{bmatrix}, \begin{bmatrix} \text{Interest} \\ \text{Rate} \end{bmatrix} \right)$$

Asset value and volatility are the only unknown quantities in these relationships and thus the two equations can be *solved* to determine the values implied by the current equity value, volatility and capital structure.

### Calculate the Distance-to-default

There are six variables that determine the default probability of a firm over some horizon, from now until time  $H$  (see Figure 8):

1. The current asset value.
2. The distribution of the asset value at time  $H$ .
3. The volatility of the future assets value at time  $H$ .
4. The level of the default point, the book value of the liabilities.
5. The expected rate of growth in the asset value over the horizon.
6. The length of the horizon,  $H$ .



### Figure 8

The first four variables, asset value, future asset distribution, asset volatility and the level of the default point, are the really critical variables. The expected growth in the asset value has little default discriminating power and the analyst defines the length of the horizon.

If the value of the assets falls below the default point, then the firm defaults. Therefore, the probability of default is the probability that the asset value will fall below the default point. This is the shaded area (EDF value) below the default point in Figure 8.

Figure 8 also illustrates the causative relationship and trade-off among the variables. This causative specification provides the analyst with a powerful and reliable framework in which they can ask what-if questions regarding the model's various inputs and examine the effects of any proposed capital restructuring. For example, the analyst can examine the effect of a large decrease in the stock price or the effects of an acquisition or merger.

If the future distribution of the distance-to-default were known, the default probability (Expected Default Frequency, or EDF value) would simply be the likelihood that the final asset value was below the default point (the shaded area in Figure 8). However, in practice, the distribution of the distance-to-default is difficult to measure. Moreover, the usual assumptions of normal or lognormal distributions cannot be used. For default measurement, the likelihood of large adverse changes in the relationship of asset value to the firm's default point is critical to the accurate determination of the default probability. These changes may come about from changes in asset value or changes in the firm's leverage. In fact, changes in asset value and changes in firm leverage may be highly correlated. Consequently, KMV first measures the distance-to-default as the number of standard deviations the asset value is away from default and then uses empirical data to determine the corresponding default probability. As discussed in a previous section, the distance-to-default is calculated as:

$$\left[ \begin{array}{c} \text{Distance} \\ \text{to Default} \end{array} \right] = \frac{\left[ \begin{array}{c} \text{Market Value} \\ \text{of Assets} \end{array} \right] - \left[ \begin{array}{c} \text{Default} \\ \text{Point} \end{array} \right]}{\left[ \begin{array}{c} \text{Market Value} \\ \text{of Assets} \end{array} \right] \left[ \begin{array}{c} \text{Asset} \\ \text{Volatility} \end{array} \right]},$$

and is marked as DD in Figure 8.

### Calculate the Default Probability

We obtain the relationship between distance-to-default and default probability from data on historical default and bankruptcy frequencies. Our database includes over 250,000 company-years of data and over 4,700 incidents of default or bankruptcy. From this data, a lookup or frequency table can be generated which relates the likelihood of default to various levels of distance-to-default.

For example, assume that we are interested in determining the default probability over the next year for a firm that is 7 standard deviations away from default. To determine this EDF value, we query the default history for the proportion of the firms, seven standard deviations away from default that defaulted over the next year. The answer is about 5 basis points (bp), 0.05%, or an equivalent rating of AA .

We have tested the relationship between distance-to-default and default frequency for industry, size, time and other effects and have found that the relationship is constant across all of these variables. This is not to say that there are no differences in default rates across industry, time, and size but only that it appears that these differences are captured by the distance-to-default measure. Our studies of international default rates are continuing but the preliminary results of studies by KMV and some of its clients indicate that the relationship is also invariant across countries and regions.

### Putting it all Together

In summary, there are three steps required to calculate an EDF credit measure: (1) estimate the current market value and volatility of the firm's assets, (2) determine how far the firm is from default, its distance-to-default and (3) scale the distance-to-default to a probability. For example, consider Philip Morris Companies Inc., which, at the end of April 2001, had a one-year EDF of 25 bp, close to the median EDF of firms with a A rating. Table 1 illustrates the relevant values and calculations for the EDF.

Variable	Value	Notes
Market value of equity	\$ 110,688 MM <sup>1</sup>	(Share Price) X (Shares Outstanding).
Book Liabilities	\$ 64,062 MM	Balance sheet.
Market value of assets	\$ 170,558 MM	Option-pricing model.
Asset volatility	21%	Option-pricing model.
Default point	\$ 47,499 MM	Liabilities payable within one year.
Distance-to-default	3.5	Ratio: $\frac{170 - 47}{170 \times 21\%}$ (In this example we ignore the growth in the asset value between now and the end of the year.)
EDF (one year)	25 bp	Empirical mapping between distance-to-default and default frequency.

<sup>1</sup> million



## A Closer Look at Calculating EDF Credit Measures

Merton's general derivative pricing model was the genesis for understanding the link between the market value of the firm's assets and the market value of its equity. It is possible to use the Black-Scholes (BS) option-pricing model, as a special case of Merton's model, to illustrate some of the technical details of estimating EDF values. The BS model is too restrictive to use in practice but is widely understood and provides a useful framework to review the issues involved. As explained before, KMV actually implements the VK model to calculate KMV's EDF credit measure. This section works an example of the calculation of an EDF value using the BS option-pricing model. The section also discusses some of the important issues that arise in practice and, where necessary, highlights the limitations of the BS model in this context.

Equity has the residual claim on the assets after all other obligations have been met. It also has limited liability. A call option on the underlying assets has the same properties. The holder of a call option on the assets has a claim on the assets after meeting the strike price of the option. In this case the strike of the call option is equal to the book value of the firm's liabilities. If the value of the assets is insufficient to meet the liabilities of the firm then the shareholders, holders of the call option, will not exercise their option and will leave the firm to its creditors.

We exploit the option nature of equity to derive the market value and volatility of the firm's underlying assets implied by the equity's market value. In particular, we solve backwards from the option price and option price volatility for the implied asset value and asset volatility.

To introduce the notation, recall that the BS model posits that the market value of the firm's underlying assets follows the following stochastic process:

$$dV_A = \mu V_A dt + \sigma_A V_A dz \quad (1)$$

where

$V_A, dV_A$  are the firm's asset value and change in asset value,

$\mu, \sigma_A$  are the firm's asset value drift rate and volatility, and

$dz$  is a Wiener process.

The BS model allows only two types of liabilities, a single class of debt and a single class of equity. If  $X$  is the book value of the debt which is due at time  $T$  then the market value of equity and the market value of assets are related by the following expression:

$$V_E = V_A N(d1) - e^{-rT} X N(d2) \quad (2)$$

where

$V_E$  is the market value of the firm's equity,

$$d1 = \frac{\ln\left(\frac{V_A}{X}\right) + \left(r + \frac{\sigma_A^2}{2}\right)T}{\sigma\sqrt{T}},$$

$$d2 = d1 - \sigma_A\sqrt{T}, \text{ and}$$

$r$  is the risk free interest rate.

It is straightforward to show that equity and asset volatility are related by the following expression:

$$\sigma_E = \frac{V_A}{V_E} \Delta \sigma_A \tag{3}$$

where

$\sigma_E$  is the volatility of the firm's equity, and

$\Delta$  is the hedge ratio,  $N(d1)$ , from (2).

Consider the example of a firm with a market capitalization of \$3 bn, an equity volatility of 40% per annum and total liabilities of \$10 bn. The asset value and volatility implied by the equity value, equity volatility and liabilities are calculated by solving the call price and volatility equations, (2) and (3), simultaneously. In this case<sup>2</sup> the implied market value of the firm's assets is \$12.511 bn, and the implied asset volatility is 9.6%.

In practice it is important to use a more general option-pricing relationship as characterized by the VK model that allows for a more detailed specification of the liabilities and that models equity as perpetuity. KMV's EDF credit measure currently incorporates five classes of liabilities, short-term, long-term, convertible, preferred equity, and common equity.

The model linking equity and asset volatility given by (3) holds only instantaneously. In practice the market leverage moves around far too much for (3) to provide reasonable results. Worse yet, the model biases the probabilities in precisely the wrong direction. For example, if the market leverage is decreasing quickly then (3) will tend to overestimate the asset volatility and thus the default probability will be overstated as the firm's credit risk improves. Conversely, if the market leverage is increasing rapidly then (3) will underestimate the asset volatility and thus the default probability will be understated as the firm's credit risk deteriorates. The net result is that default probabilities calculated in this manner provide little discriminatory power.

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<sup>2</sup> All liabilities are assumed to be due in one year,  $T = 1$ , and the interest rate  $r$  is assumed to be 5%.

Instead of using the instantaneous relationship given by (3), KMV's EDF credit measure is produced using a more complex iterative procedure to solve for the asset volatility. The procedure uses an initial guess of the volatility to determine the asset value and to *de-lever* the equity returns. The volatility of the resulting asset returns is used as the input to the next iteration of the procedure that in turn determines a new set of asset values and hence a new series of asset returns. The procedure continues in this manner until it converges. This usually takes no more than a handful of iterations if a reasonable starting point is used. In addition, the asset volatility derived above is combined in a Bayesian manner with country, industry and size averages to produce a more predictive estimate of the firm's asset volatility.

The probability of default is the probability that the market value of the firm's assets will be less than the book value of the firm's liabilities by the time the debt matures. That is:

$$p_t = \Pr[V_A^t \leq X_t \mid V_A^0 = V_A] = \Pr[\ln V_A^t \leq \ln X_t \mid V_A^0 = V_A] \quad (4)$$

where

$p_t$  is the probability of default by time  $t$

$V_A^t$  is the market value of the firm's assets at time  $t$ , and

$X_t$  is the book value of the firm's liabilities due at time  $t$ .

The change in the value of the firm's assets is described by (1) and thus the value at time  $t$ ,  $V_A^t$ , given that the value at time 0 is  $V_A$ , is:

$$\ln V_A^t = \ln V_A + \left( \mu - \frac{\sigma_A^2}{2} \right) t + \sigma_A \sqrt{t} \varepsilon \quad (5)$$

where

$\mu$  is the expected return on the firm's asset, and

$\varepsilon$  is the random component of the firm's return.

The relationship given by equation (5) describes the evolution in the asset value path that is shown in Figure 8. Combining (4) and (5), we can write the probability of default as:

$$p_t = \Pr \left[ \ln V_A + \left( \mu - \frac{\sigma_A^2}{2} \right) t + \sigma_A \sqrt{t} \varepsilon \leq X_t \right], \quad (6)$$

and after rearranging:

$$p_t = \Pr \left[ -\frac{\ln \frac{V_A}{X_t} + \left( \mu - \frac{\sigma_A^2}{2} \right) t}{\sigma_A \sqrt{t}} \geq \varepsilon \right] \quad (7)$$

The BS model assumes that the random component of the firm's asset returns is Normally distributed,  $\varepsilon \sim N(0,1)$  and as a result we can define the default probability in terms of the cumulative Normal distribution:

$$p_t = N \left[ -\frac{\ln \frac{V_A}{X_t} + \left( \mu - \frac{\sigma_A^2}{2} \right) t}{\sigma_A \sqrt{t}} \right], \quad (8)$$

Recall that the distance-to-default is simply the number of standard deviations that the firm is away from default and thus in the BS world is given by:

$$DD = \frac{\ln \frac{V_A}{X_t} + \left( \mu - \frac{\sigma_A^2}{2} \right) t}{\sigma_A \sqrt{t}} \quad (9)$$

Continuing with our example, assume that the expected return on the assets,  $\mu$ , is equal to 7% and that we are interested in calculating the one-year default probability. The distance-to-default,  $DD$ , in this case<sup>3</sup> is 3.0, and the corresponding default probability from (8) is 13 bp.

In practice, we need to adjust the distance-to-default to include not only the increases in the asset value given by the rate but also adjust for any cash outflows to service debt, dividends and so on. In addition, the Normal distribution is a very poor choice to define the probability of default. There are several reasons for this but the most important is the fact that the default point is in reality also a random variable. That is, we have assumed that the default point is described by the firm's liabilities and amortization schedule. Of course we know that this is not true. In particular, firms will often adjust their liabilities as they near default. It is common to observe the liabilities of commercial and industrial firms increase as they near default while the liabilities of financial institutions often decrease as they approach default. The difference is usually just a reflection of the liquidity in the firm's assets and thus their ability to adjust their leverage as they encounter difficulties.

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<sup>3</sup> The distance-to-default is calculated by equation (9),  $DD = \frac{\ln \frac{12.5116}{10} + \left( 0.07 - \frac{0.0092}{2} \right)}{0.0961}$ .

Unfortunately ex ante we are unable to specify the behavior of the liabilities and thus the uncertainty in the adjustments in the liabilities must be captured elsewhere. We include this uncertainty in the mapping of distance-to-default to the EDF credit measure. The resulting empirical distribution of default rates has much wider tails than the Normal distribution. For example, a distance-to-default of four, four standard deviations, maps to a default rate of around 100 bp. The equivalent probability from the Normal distribution is essentially zero.

## Calculating Long-Term EDF Credit Measures

The extension of the model to longer terms is straightforward. The default point, asset volatility and expected asset value are calculated as before except they take into account the longer horizon, see Figure 9. For example, suppose we are interested in calculating the EDF value for a three-year horizon. Over the three years, we can expect that the default point will increase as a result of the amortization of long-term debt. This is a conservative assumption that all long-term debt is refinanced short-term. We could just as easily model the asset value decreasing as the debt is paid down but in practice debt is usually refinanced. In any case it really doesn't matter, whether the assets go down by the amount of the amortization or the default point increases by the same amount, the net effect on the default point is the same.

In addition to the default point changing, as we extend the horizon the future expected asset value is increasing as is our uncertainty regarding its actual future value. The expected asset value increases at the expected growth rate and the total asset volatility increases proportionally with the square root of time<sup>4</sup>.

The distance-to-default is therefore calculated using the relevant three-year asset value, asset volatility and default point. The scaling of the default probability again uses the empirical default distribution mapping three-year distance-to-defaults with the cumulative default probability to three years. That is, the mapping answers the question, what proportion of firms with this three-year distance-to-default actually default within three years. The answer to this question is the three-year cumulative default probability. EDF values are annual default probabilities and the three-year EDF value is calculated as the equivalent average annual default probability<sup>5</sup>. For example, suppose the three-year cumulative probability is 250 bp then the three-year EDF value is 84 bp.

<sup>4</sup> The asset variance is additive and therefore increases linearly with time. The asset volatility is the square root of the variance and therefore increases with the square root of time.

<sup>5</sup> The EDF is calculated from the cumulative default probability using survival rates. For example, the three-year cumulative probability of default and the three-year EDF are related by the following expression:

$$1 - CEDF_3 = (1 - EDF_3)^3 .$$

The probability of not defaulting within three years,  $1 - CEDF_3$ , and the average annual probability of not defaulting,  $1 - EDF_3$ .

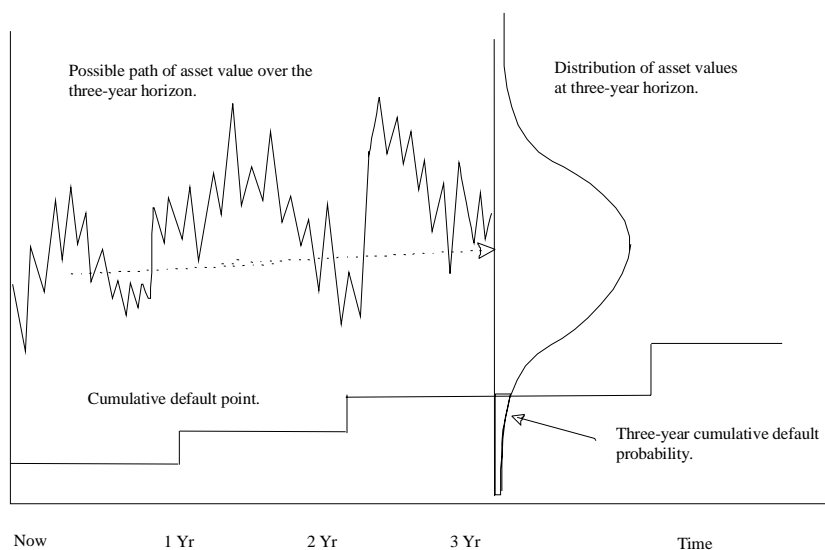


Figure 9

## Some Frequently Asked Questions About KMV's EDF Credit Measures

*How does the model deal with off-balance-sheet liabilities?*

This is a critical question for many firms, particularly financial institutions where these liabilities can obviously be quite significant. Fortunately, the model is surprisingly robust to the precise level of the liabilities.

For example, consider the firm used previously in our BS example. Assume that in addition to the \$10 bn in liabilities the firm has a further \$5 bn in off-balance-sheet commitments. That is, the true default point is actually \$15 bn not \$10 bn. The *actual* EDF value of the firm can therefore be calculated using the BS model as follows. The firm's market capitalization remains \$3 bn, and its equity volatility is still 40% per annum. The implied asset value and volatility, with liabilities of \$15 bn, are calculated again by solving the call price and volatility equations, (2) and (3), simultaneously. In this case<sup>6</sup> the implied market value of the firm's assets is \$17.267 bn, and the implied asset volatility is 6.9%. The asset value is about \$5 bn higher and the asset volatility is lower reflecting the higher leverage of the firm. (Recall the equity volatility was kept the same but we increased the leverage, as a result the implied asset volatility must be lower.) The corresponding distance-to-default is slightly higher than 3.0 and the implied EDF value is still approximately 13 bp.

<sup>6</sup> All liabilities are assumed to be due in one year,  $T = 1$ , and the interest rate  $r$  is assumed to be 5%.

Obviously if you have more complete or up-to-date information on the firm's liability structure it should be used in the model. Credit Monitor includes an add-on product called EDF Calculator™ (EDFCalc™) that enables the user to enter a more complete, or more recent, statement of the firm's liabilities.

*Does the model incorporate the possibility of large changes in the market value of the firm?*

Yes. In addition to incorporating the uncertainty in the liability structure of the firm, the empirical distance-to-default to EDF value distribution captures the possibility of large jumps, up or down, in the firm's market value. The empirical distribution includes data from several serious market downturns including the crash of October 1987.

*Can the model be used to simulate market downturns or crashes?*

The EDF credit measure already includes the effects of market downturns and crashes weighted by their appropriate probabilities. However, it is quite straightforward to ask questions such as, in the event of a 30% drop in the market what will be the effect on a firm's, or a portfolio of firms', EDF values? The effect of a market downturn on the equity value of any particular firm can be estimated using the firm's equity beta.

$$\Delta V_E = \beta_E \Delta V_m \quad (10)$$

*Should EDF values be averaged or smoothed to remove their variation over time?*

No. It is certainly true that the EDF value of a firm can vary over time but these variations are reflecting changes in credit quality as perceived by the equity market. Therefore any smoothing or averaging is simply masking the signals from the market.

The volatility in an EDF credit measure over time can pose problems for some bank's credit processes where the EDF value directly determines the grade. However, this issue is usually simply overcome by determining actions by range of EDF values. That is, action triggers are attached to grades that are defined in terms of EDF value ranges. As a result, small, economically insignificant, movements within a grade do not trigger any action and movements between grades trigger an appropriate review.

A related question is whether or not there is any trend information in EDF value. There is not. EDF values are driven by market prices and thus are directly analogous to prices. If there isn't any trend information in the equity price there isn't any in the EDF credit measure.

*Why isn't information from the bond or credit derivatives market included?*

There is a whole class of models, usually called reduced-form models, that relate credit spreads and default probabilities. Our experience implementing these approaches has not been successful to date. There is nothing wrong with the models per se, indeed in theory they hold the promise of some advantages over the causal model described in this paper. However, the data required to calibrate and implement reduced-form models is not yet widely available. In most cases credit risk simply is not as actively and cleanly traded as equities at the moment.

This situation will undoubtedly change as the credit derivative and other markets grow but to date we have not found credit spread information to be of sufficient quality to support the estimation of individual level default probabilities.

To date, the most successful use of credit spread data that we are aware of has been in the cross-sectional<sup>7</sup> estimation of credit spread curves. These curves describe the typical market spread for a given level of credit quality.

*Are the default probabilities applicable across countries and industries?*

The distance-to-default measure incorporates many of the idiosyncrasies of different countries and industries. For example, the business risk, as measured by the asset volatility, varies for a given industry across countries. Volatilities tend to be the lowest in Europe and the highest in the United States with Asian countries usually in-between. With the exception of the difficulty posed by differing accounting standards, the default point can be measured appropriately for each firm regardless of its country of incorporation. The different economic prospects for countries are obviously captured by the individual equity and asset valuations. As a result, we believe that the distance-to-default captures most of the relevant inter-country differences in default risk. However, the question does remain whether differences in bankruptcy codes, culture and so on may result in different default rates for a given distance-to-default. That is, is the distance-to-default to EDF value mapping constant across countries?

KMV's empirical default distribution that is used to map distance-to-default to EDF credit measures is built from publicly listed defaults in the United States. As a result its translation to other countries should be questioned. However, we believe that the default probabilities resulting from this US based mapping are good measures of economic default risk. That is, it is possible as a result of political or other intervention that a firm will be saved from default in say Europe when in the US it might have been left to fail. However, these interventions are not free and are costly to someone, often the taxpayer, or in the case of Asia, more commonly the shareholders of related firms. It seems unwise to us to measure default risk incorporating a reliance on the uneconomic behavior of another group. The uneconomic behavior may well continue but it certainly seems unwise to institutionalize it in a measure of default risk. We do not believe it is possible to reliably model uneconomic behavior and thus we aim to provide a hard economic measure of default and allow analysts to factor in their own measure of implied government or other support.

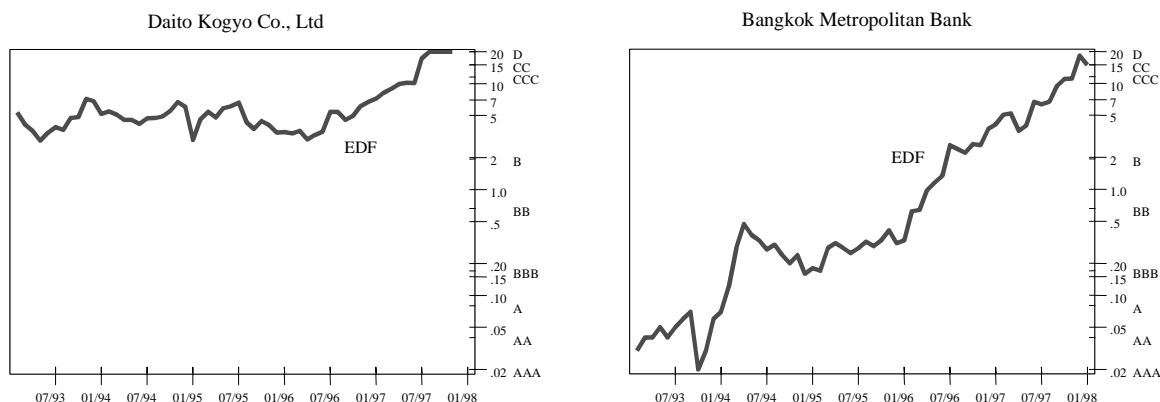
Philosophy aside, our experience with EDF values internationally has been very good. Over half the users of KMV's EDF credit measures operate outside of the US. During the Asian credit problems in 1997, the model performed extremely well, see examples in Figure 10. More recent credit difficulties in a number of countries around the world have been foreshadowed by appropriate increases in the EDF credit measures of firms in distress. The model has also been tested anecdotally in most European countries. We continue to collect default data

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<sup>7</sup> Cross-sectional in this context means combining data from many different firms and issues. This is in contrast to the problem tackled in this paper—the estimation of the default probability for each individual firm.



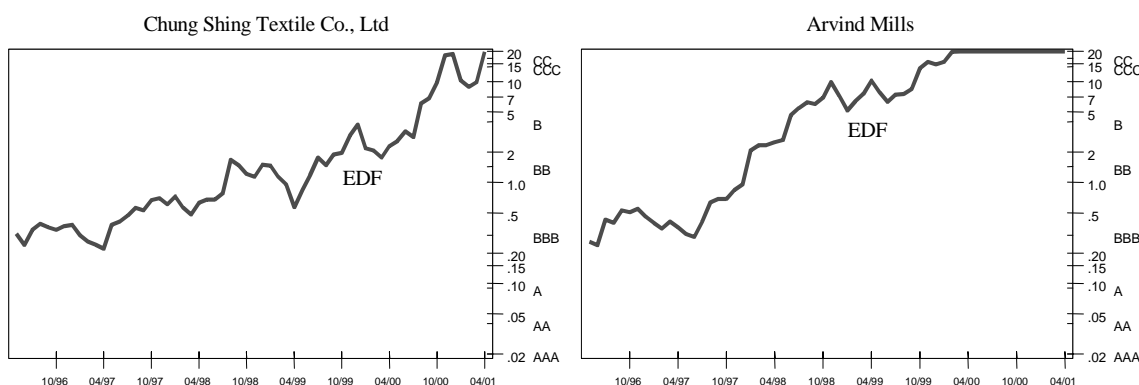
internationally and continue to release studies of the KMV EDF credit measure's performance as these data accumulate.



**Daito Kogyo, a Japanese construction company, defaulted in July 1997.**

**Bangkok Metropolitan Bank (Thailand) was taken over in January 1998.**

**Figure 10**



**Chung Shing Textile Co., Ltd, a Taiwanese Thread Mill company, defaulted in April 2001.**

**Arvind Mills, an Indonesian Fabric Mill, defaulted in February 2001.**

**Figure 10**

*Does the EDF credit measure contain any measure of country (translation) risk?*

It is likely there is some measure of country risk impounded in the EDF credit measure; but exactly how much we don't know. The calculations for KMV's EDF credit measures are done in the firm's local currency and therefore the country risk measure comes from the discount in the local equity price as a result of international investors concerns regarding the convertibility of

the currency. Obviously it is impossible to separate this influence from all the others and as a result we do not know how much country risk, if any, is present in the EDF value. The amount is likely to vary by country as a function of the accessibility of their equity market and on the interest of international investors.

*How well does the EDF credit measure work on thinly traded or closely held firms?*

Surprisingly well. This question often arises in connection with efficient markets and the culture of Anglo-Saxon markets. While the results of the international default studies will have to speak for themselves we already actually know quite a lot about the performance of the model in thinly traded markets.

Our experience with these firms is drawn from the bottom end of the US equity market where the companies are smaller and less actively traded than most of the firms on international exchanges. Our coverage of the US is almost total and the bottom 2,000 or so companies have market caps of less than USD 20 m, (almost 1,000 have a market capitalization of less than USD 7 m). Most of these companies are not even listed on an exchange and are traded instead over the counter. Obviously trading in these companies is going to be thin and many are likely to be very closely held.

This is also a group of firms that default often and thus we have a large body of evidence on the model's performance on these firms. It is very good. It appears that it doesn't seem to take many economically motivated investors to move the equity price to reflect the risk of the firm.

*Does the model assume that the equity market is efficient?*

No. The efficiency of a market usually refers to the degree to which the current price reflects all the relevant information about a firm's value. While we do not necessarily assume that the price reflects all the relevant information about a firm we do know that it is difficult to consistently beat the market. For example, over 90% of managed funds were unable to outperform the market in 1998. That is, it is difficult to pick stocks consistently and difficult to know when the market is under- or overvaluing a firm. The market reflects a summary of many investors forecasts and it is unusual if any one individual's, or committee's, forecast is better. Consequently, we believe that the best source of information regarding the value of a firm is the market.

The market though can be caught by surprise as in Figure 11. Koninklijke KPN NV, a Dutch telephone company, caught the market and rating agencies by surprise although some credit analysts undoubtedly worried that it was overvalued.

Fraud is often the cause of extremely large and sudden changes in credit quality. For example, in Figure 12 it is not hard to spot when the announcement regarding the improprieties in the reporting of Mercury Finance's assets was made.

However, most of the time the market will be well aware of problems, or opportunities, and this information will be fairly reflected in the EDF value, see Figure 13.

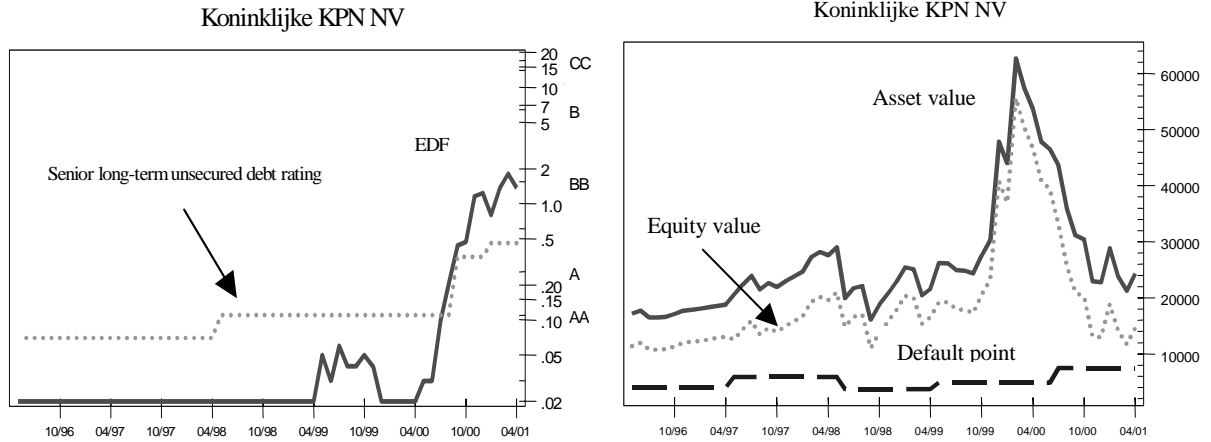


Figure 11

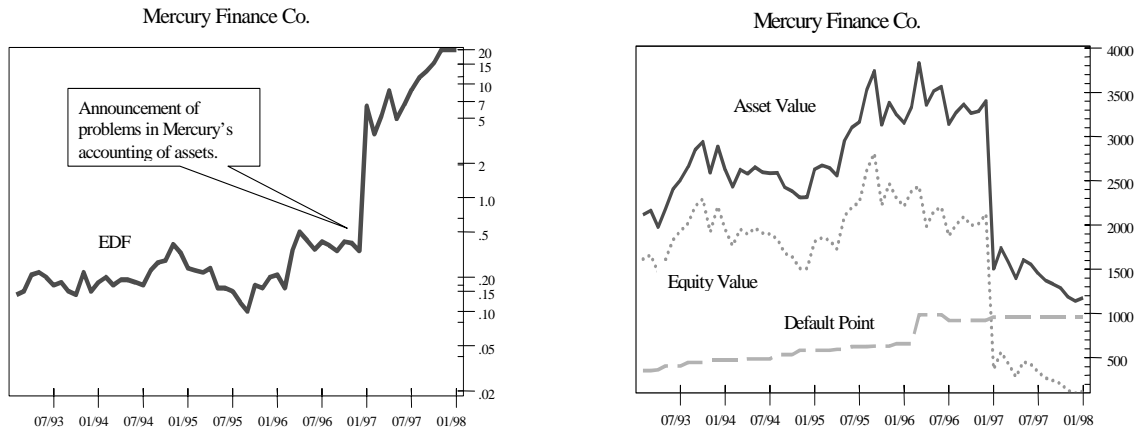
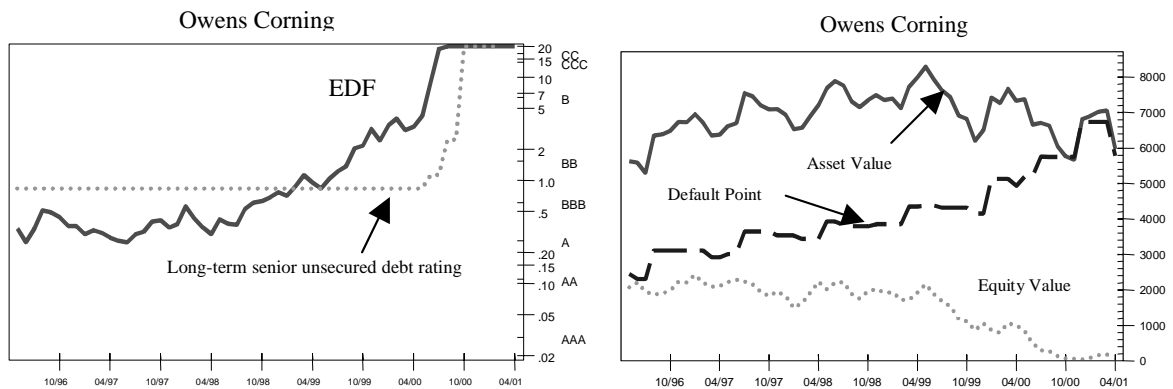


Figure 12



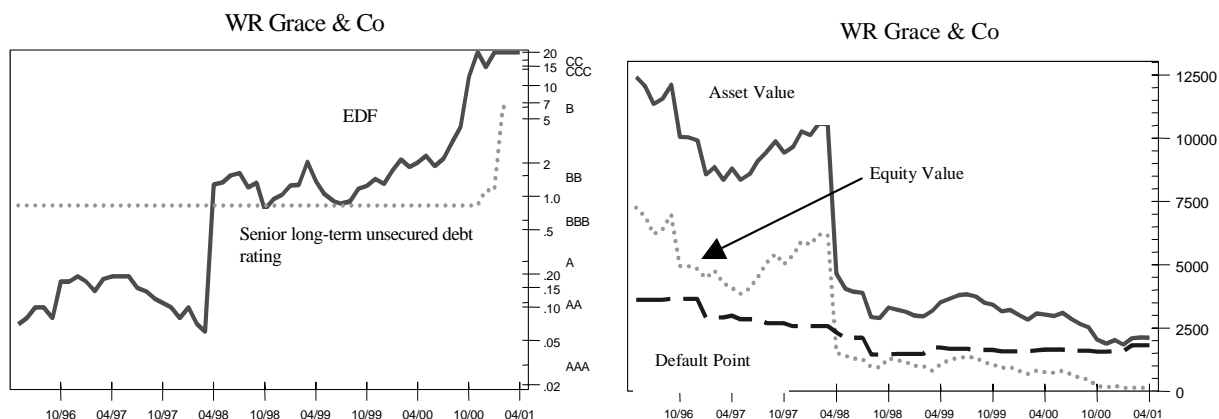
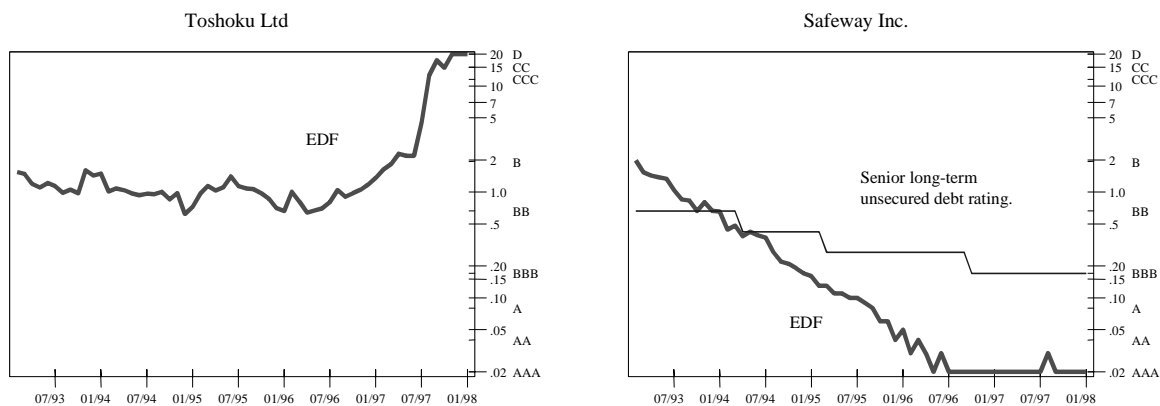


Figure 13

Owens Corning, an American Construction Materials company, failed in November 2000. The equity value of WR Grace, a US Chemical Products company, dropped significantly in May 1998. The company failed in October 2000.



Toshoku, a Japanese food trading company, failed in December 1997. The equity value of Safeway, a US food retailer, has increased by \$12.5 bn over the last four years while its liabilities have increased by only \$1.5 bn.

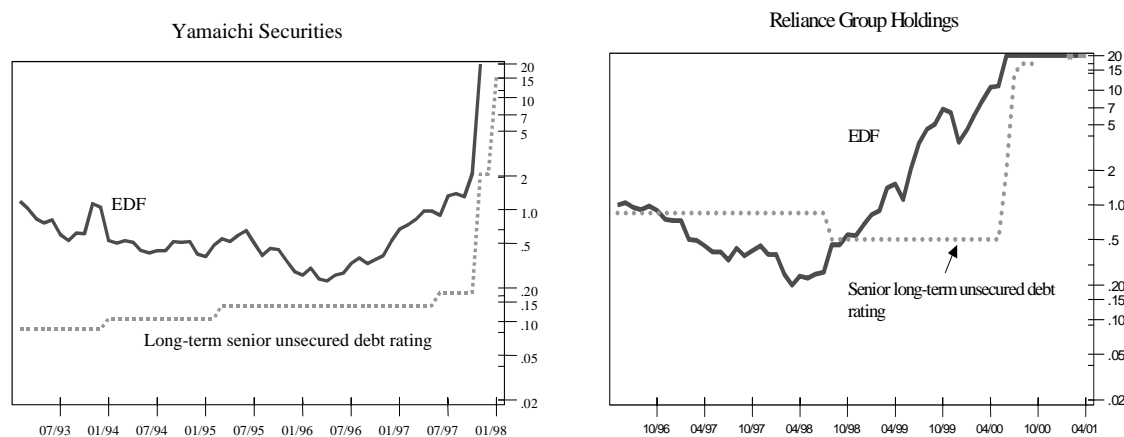
Figure 13

*How well does the model work on financial institutions?*

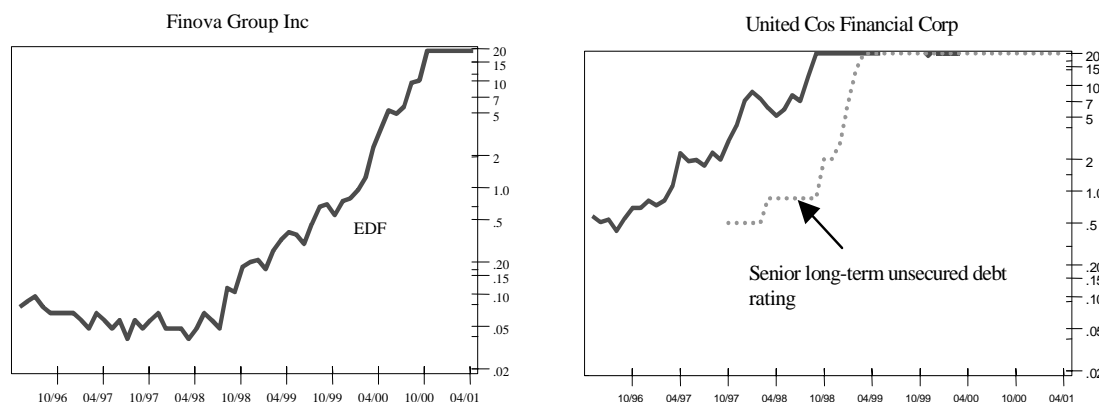
The credit risk of financial institutions is notoriously difficult to assess. Financial institutions are typically very opaque and thus judging the quality of their assets and determining the extent of their liabilities is almost always very difficult. In addition, the liquidity of their assets means that their true size is sometimes difficult to judge. The window dressing of balance sheets at reporting dates is common place.

The equity market is well aware of these issues and no doubt does better than most at sorting them out. In addition, we are fortunate that EDF values are relatively robust to the understatement of a firm's liabilities (see our earlier discussion on this issue). However, it is undoubtedly true that many financial institutions stretch this property of the model to its limits. In addition to these challenges, most financial institutions are tightly regulated and thus the appropriate definition of default may not be the point when their asset value falls below their liabilities. Unfortunately, there are very few financial institution defaults and thus testing and calibrating the model on just financial institutions is difficult.

Overall, despite these challenges we believe that the model performs very well on financial institutions, certainly better than any alternative approach that we know of. The lack of actual defaults means it is difficult for us to determine if the level of the EDF value is as precise as it is for commercial and industrial firms, but the anecdotal evidence is clear; the model provides timely and reliable early warning of financial difficulty, see Figure 14.



Yamaichi Securities defaulted during November 1997 and Reliance Group Holdings the property and casualty insurance company has been in default on its payment obligations since November 2000.



**Finova Group Inc., the financial services company filed for Chapter 11 protection in March 2001. United Companies Financial Group, the mortgage banker and Loan Company filed for bankruptcy in March 1999.**

**Figure 14**

*How does this apply to firms that do not have publicly traded equity?*

One of the themes of this paper has been that the equity value of the firm conveys a considerable amount of information regarding the firm's credit quality. When this information is not available, we are forced to fall back on peer comparisons to determine the asset value and asset volatility. We do this analysis in a companion product to Credit Monitor (CM) called the Private Firm Analyst™ (PFA). CM enabled with PFA provides the tools for calculating and analyzing an EDF value for firms without publicly traded equity. These EDF values are calculated using KMV's Private Firm Model™ (PFM)<sup>8</sup>.

PFM uses a similar framework to the VK model except that the asset value and asset volatility are estimated using financial statement data and industry and country comparables. The estimation of the asset volatility is relatively straightforward. As we have seen, size, industry and country can explain asset volatility quite well. Estimating the asset value is much more challenging. PFM uses a broad set of comparables and essentially determines an appropriate EBITDA<sup>9</sup> multiple. That is, given the set of comparables, what is a reasonable multiple to apply to the private firm's EBITDA to determine its asset value.

In spite of the obvious challenges that the absence of market data poses, overall the PFM does rather well. For example, in Figure 15 we plot the EDF values from CM and PFM along with senior unsecured debt rating for two public firms. Marvel Entertainment, a US publishing company that defaulted in December of 1996 and Ben Franklin Retail Stores, a US retail company, that defaulted in July 1997. As you would expect, at any one point in time, the

<sup>8</sup> The PFM is discussed more fully in other publications available from KMV.

<sup>9</sup> We use EBITDA, earnings before interest, taxes, depreciation and amortization, as a proxy for the firm's free cash flow.

correspondence between the public and private EDF credit measures is far from perfect. However, longitudinally the correspondence can be quite remarkable as it is in both of these cases.

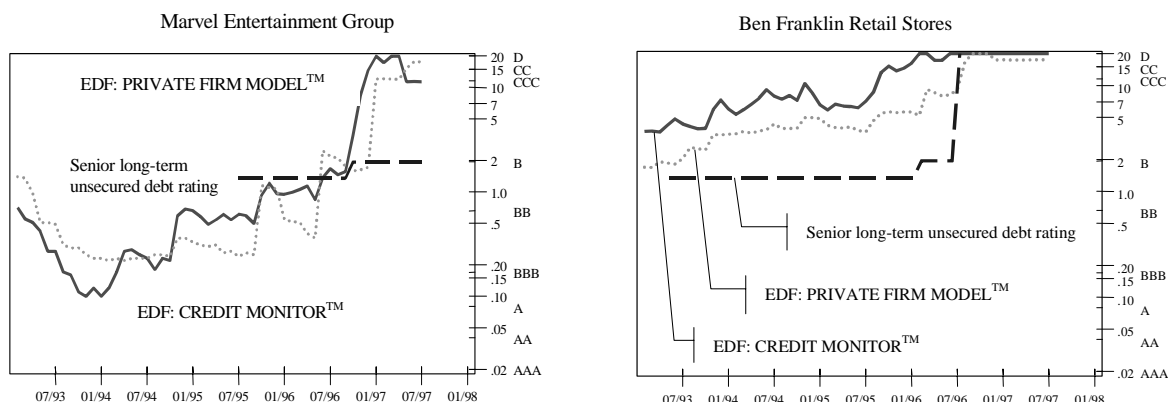
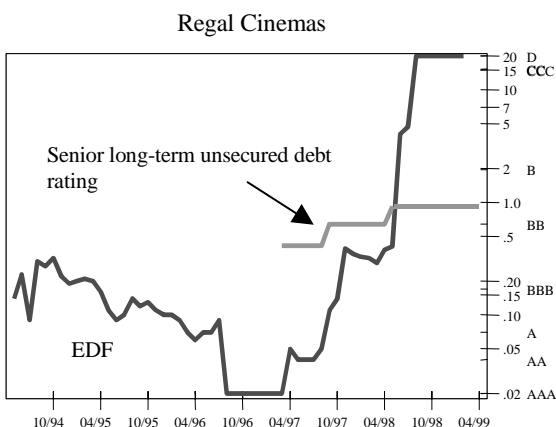


Figure 15

PFM’s performance on truly private companies has been tested extensively in the US and efforts are underway to extend this testing to Europe and Asia. There are some sectors in which the PFM does not do well at all. Most notably, it cannot be used on financial institutions. The operating cash flow for these firms is a very poor indicator of asset value.

The public market comparables tie the PFM’s EDF credit measure into the credit cycle. That is, because the EBITDA multiples adjust to reflect the current market conditions and outlook, the EDF credit measures from the PFM change over time even if the financial statements of the firm remain stable. This is obviously a key property of the model and within the limitations of financial statement data keeps the EDF values as forward looking as possible.



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**Regal Cinemas, a US Motion Pictures company, went private in May 1998 after an LBO. The Private Firm Model <sup>®</sup> EDF credit measure on the LBO debt came out at 20% in August 1998. Regal Cinemas defaulted in September 2000.**

Figure 16

## Testing the Default Measure's Performance

Determining the performance of a default measure is both a theoretical and an empirical problem. For example, what exactly do we mean by performance or predictive power? In practice, we can only hope to estimate probabilities of default. That is, we will not be able to definitively classify firms into *will default* and *will not default* categories. As a result, in assessing the performance of a model, we face the task of assessing its ability to discriminate between different levels of default risk.

For example, consider the policy of never lending to firms below a 2% EDF value, around a B rating. The benefit of this policy is that we avoid lending to firms that have a relatively high probability of default and thus avoid lending to a lot of firms that do eventually default. The cost of this policy is that we do not lend to any firms below a B rating and many of these firms, about 98%, do not default. Thus, one measure of a model's performance is the tradeoff between the defaulting firms we avoid lending to and the proportion of firms we exclude. This tradeoff is commonly called the power curve of a model.

For example, in Figure 16 we plot the power curves for EDF credit measures and the senior unsecured debt rating from a major bond rating agency. The cutoff points for the population are plotted along the horizontal axis and the proportion of defaults excluded at each cutoff point is plotted on the vertical axis. If we rank all firms by their EDF credit measures and impose a cutoff at the bottom 10%, then we avoid lending to 73% of the defaulting firms. That is, by not lending to the bottom 10% as ranked by the EDF credit measure we can avoid 73% of all defaulting firms. At a cutoff of 30% we are able to avoid lending to 97% of defaulting firms and of course if we do not lend to anybody, a cutoff of 100%, we avoid lending to all of the defaulting firms. Thus, for a given cutoff, the larger the proportion of defaults that are excluded, the more powerful is the model's ability to discriminate high default risk firms from low default risk firms.



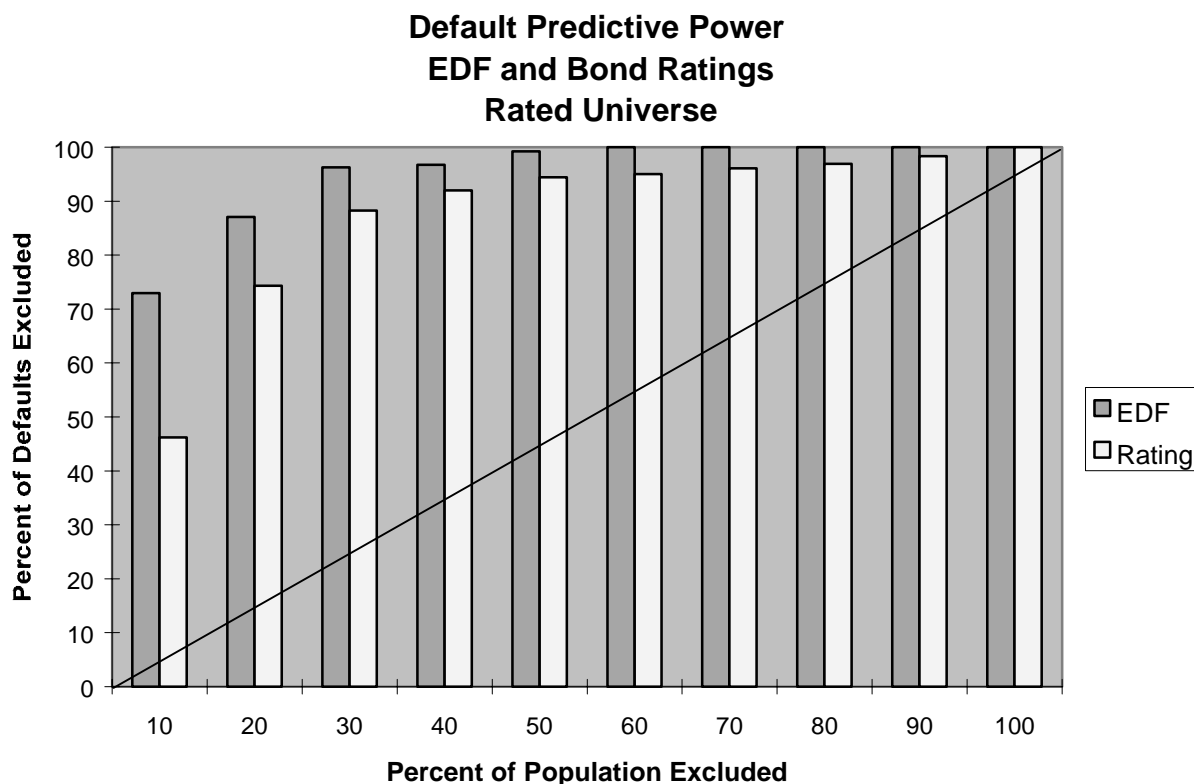
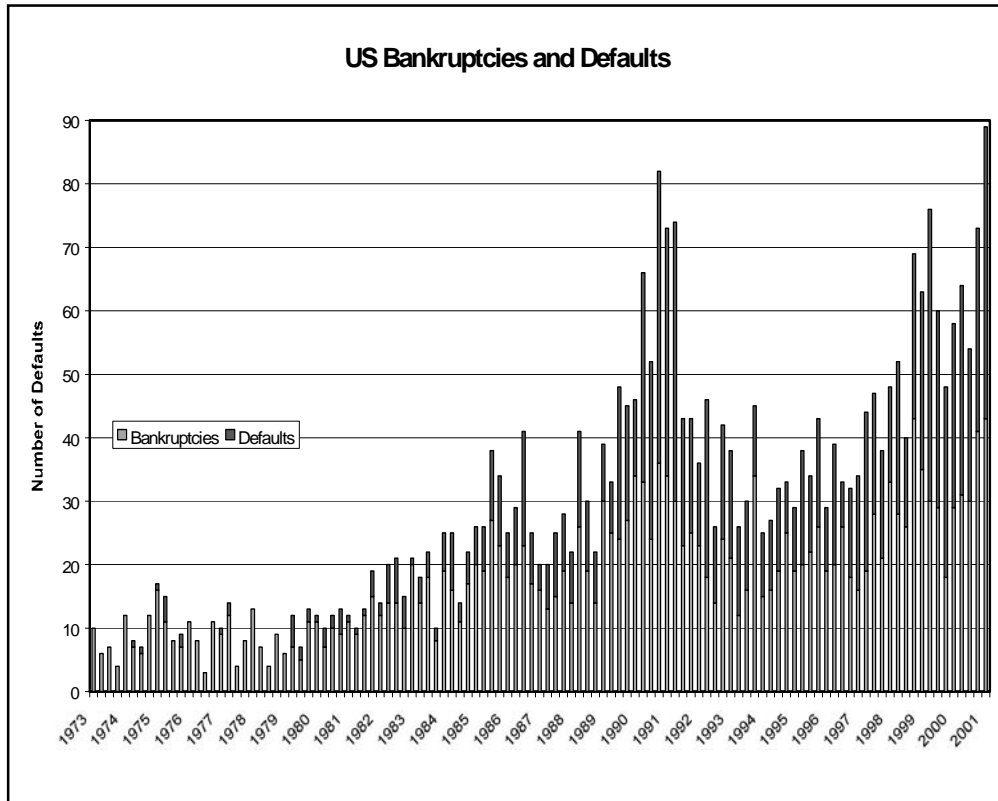


Figure 16

The overall default rate, and thus the default probability of firms, varies considerably over time. Figure 17 plots the default history for the US from 1973 through 2001. The chart shows that as a general rule of thumb we can expect the default rate to double or triple between the high and low of the credit cycle. Thus, an effective measure of default risk cannot average default rates over time; instead, it must reflect the changes in default risk over time. Because KMV's EDF credit measure incorporates asset values based on information from the equity market, it naturally reflects the credit cycle in a forward-looking manner. For example, Figure 18 shows the median EDF value for US A, BBB, BB and B rated firms from April 1996 through April 2001, and Figure 19 shows the EDF value quartiles for financial institutions in Korea and Thailand from February 1993 through January 1997.

Figure 17



Median EDF A, BBB, BB and B Rated US Corporates

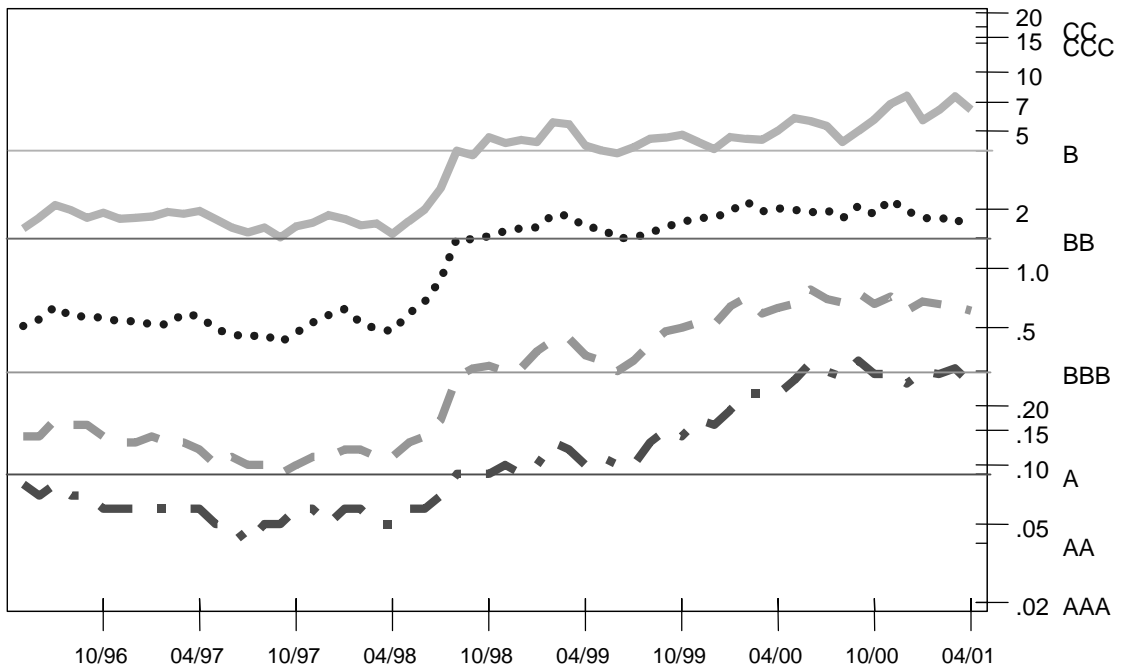


Figure 18

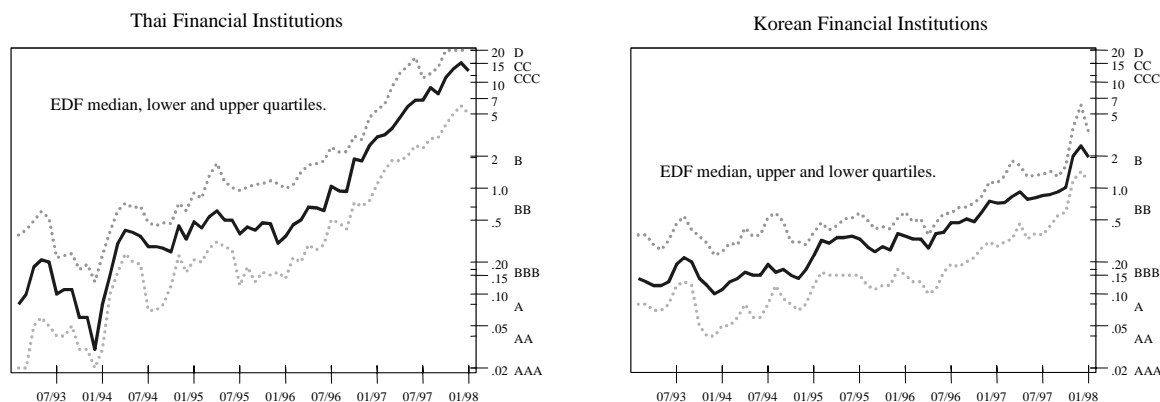


Figure 19

At the individual firm level, the model's ability to reflect the current credit risk of a firm can be assessed by observing the change in the EDF value of a firm as it approaches default. Figure 20 plots the medians and quartiles of the EDF values for five years prior to the dates of default for rated companies. Default dates are aligned to the right such that the time moving to the left indicates years prior to default. EDF values are plotted along the vertical axis. The level of EDF values is sloping upward, towards increasing levels of default risk, as the date of default draws closer. Moreover, the slope increases as the date of default approaches.

Five years prior to default, the median EDF value of defaulting companies is approximately 1%, around BB. One year prior to default the median has increased to over 6%. During the time of this sample, the median EDF value for all rated companies, both default and non-default, was around 0.16%. (The median and percentiles for the rated universe are the straight lines running parallel to the horizontal axis at the bottom of the chart.) Two years prior to default, the lower quartile of EDF values (the riskiest 25%) of the defaulting firms breaks through the upper quartile of the rated universe (the safest 25% as measured by the rating agency). Thus, two full years prior to default 75% of the defaulting firms had EDF values in the bottom quartile of the universe.

There is no single measure of performance for default measures such as KMV's EDF credit measure. Performance must be measured along several dimensions including discrimination power, ability to adjust to the credit cycle and the ability to quickly reflect any deterioration in credit quality. The EDF value generated from the equity market and financial statement information of a firm does all of these things well. The dynamics of the EDF credit measure come mostly from the dynamics of the equity value. It is simply very hard to hold the equity price of a firm up as it heads towards default. The ability to discriminate between high and low default risks comes from the distance-to-default ratio. This key ratio compares the firm's net worth to its volatility and thus embodies all of the key elements of default risk. Moreover, because the net worth is based on values from the equity market, it is both a timely and superior estimate of the firm's value.

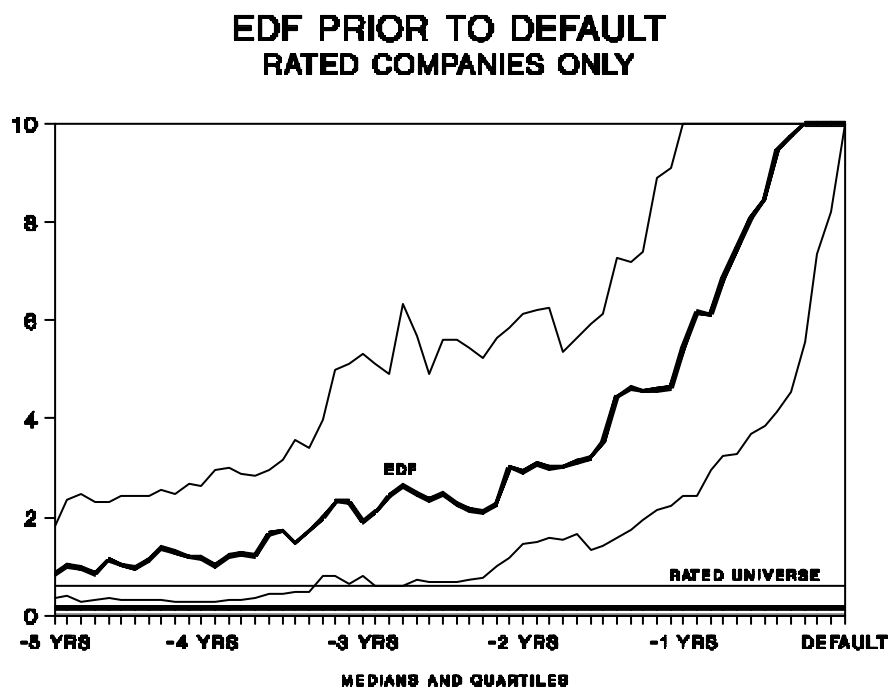


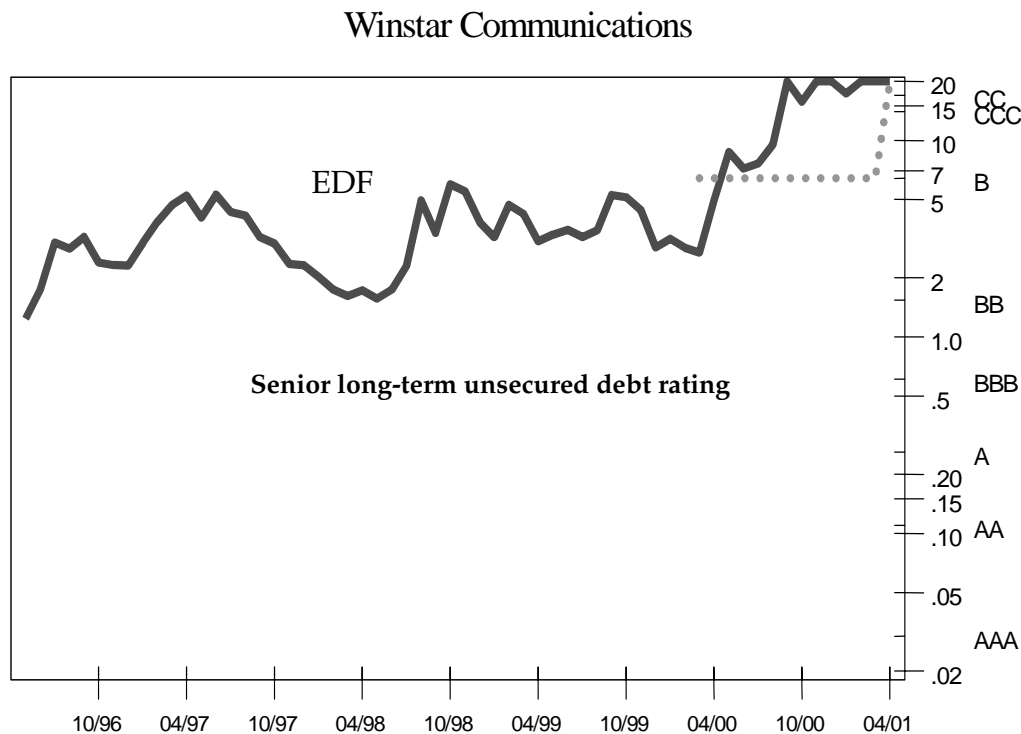
Figure 20

## Summary and Discussion

A three-step process is used to calculate KMV's EDF<sup>TM</sup> credit measure:

1. Estimate the market value and volatility of the firm's assets,
2. Calculate the distance-to-default, the number of standard deviations the firm is away from default, and
3. Scale the distance-to-default to an expected default frequency (EDF) using an empirical default distribution.

Because EDF credit measures are based on market prices they are forward looking and reflect the current position in the credit cycle. They are a timely and reliable measure of credit quality. As a final example of the forward-looking strength of EDF values, Figure 21 shows Winstar Communications, which filed for bankruptcy in April of 2001. (Venture Stores was presented in our first example, Figure 1 on page 2.) The first sign of a serious deterioration in the credit quality is in August 1998 when the EDF jumps from 1.7% to 2.3% (BB to B). The EDF value climbed as high as 6% in October 1998, recovering a little as they secured additional financing before finally reaching 20% (D) in September 2000, eight months prior to filing for bankruptcy.



**Figure 21**

EDF credit measures are an effective tool in any institution's credit process. Accurate and timely information from the equity market provides a continuous credit monitoring process that is difficult and expensive to duplicate using traditional credit analysis. Annual reviews and other traditional credit processes cannot maintain the same degree of vigilance that EDF values calculated on a monthly or a daily basis can provide. Continuous monitoring is the only effective early warning protection against deteriorating credit quality. EDF values are also often used to help the focus of the efforts of traditional credit processes. They provide a cost-effective method to screen credits quickly and to focus credit analysis where it can add the most value. Further, because EDF values are real probabilities, they are the key data items in many institutions' provisioning, valuation and performance measurement calculations.