The Information in Long-Maturity Forward Rates

By EUGENE F. FAMA AND ROBERT R. BLISS*

Current 1-year forward rates on 1- to 5-year U.S. Treasury bonds are information about the current term structure of 1-year expected returns on the bonds, and forward rates track variation through time in 1-year expected returns. More interesting, 1-year forward rates forecast changes in the 1-year interest rate 2- to 4-years ahead, and forecast power increases with the forecast horizon. We attribute this forecast power to a mean-reverting tendency in the 1-year interest rate.

Much of the empirical work on the term structure of interest rates is concerned with two questions. (a) Do current forward rates forecast future interest rates? (b) Do current forward rates have information about the structure of current expected returns on bonds with different maturities? Much of the empirical work on these questions uses U.S. Treasury bills and so is restricted to maturities less than a year. This paper studies the information in forward rates about future interest rates and current expected returns for annual U.S. Treasury maturities to 5 years.

Our results on expected bond returns are novel. Past tests typically fail to produce reliable inferences about the structure of expected returns for maturities beyond a year. (See, for example, Reuben Kessel, 1965; J. Huston McCulloch, 1975; and Fama, 1984b.) Using the regression approach in Fama (1984a, 1986), we are able to infer that 1-year expected returns for maturities to 5 years, measured net of the interest rate on a 1-year bond, vary through time. These expected premiums swing from positive to negative, however. On average, the term structure of 1-year expected returns on 1- to 5-year Treasury bonds is flat.

Differences in expected returns are usually interpreted as rewards for risk. In this view, our evidence that the ordering of expected returns across maturities changes through time implies changes in the ordering of risks. This behavior of expected returns is inconsistent with simple term structure models, like the liquidity preference hypothesis of John Hicks (1946) in which expected returns always increase with maturity. The evidence poses an interesting challenge to models like those of Robert Merton (1973), John Long (1974), Douglas Breeden (1979), and John Cox et al. (1985), that allow time-varying expected returns.

Our results on the forecast power of forward rates are also novel. Previous tests find little evidence that forward rates can forecast future interest rates. For example, Michael Hamburger and E. N. Platt (1975) and Robert Shiller et al. (1983) conclude that forward rates have no forecast power. Fama (1984a) finds some power to forecast 1-month interest rates 1 month ahead. We confirm that forward rate forecasts of near-term changes in interest rates are poor. When the forecast horizon is extended, however, forecast power increases. The 1-year forward rate calculated from the prices of 4- and 5-year bonds explains 48 percent of the variance of the change in the 1-year interest rate 4 years ahead. We argue that this forecast power is largely due to a slow mean-reverting tendency in interest rates which is more apparent over longer horizons.

The hypothesis that interest rates are mean reverting is prominent in old and new mod-

*Graduate School of Business, University of Chicago, Chicago IL 60637. The helpful comments of John Cochrane, Bradford Cornell, Wayne Ferson, Kenneth French, Merton Miller, Richard Roll, and two referees are gratefully acknowledged. This research is supported by the National Science Foundation (Fama) and the Center for Research in Security Prices (Bliss).

els of the term structure, for example, F. A. Lutz (1940) and Cox et al. Unlike other recent work (for example, Charles Nelson and Charles Plosser, 1982, and Fama and Michael Gibbons, 1984), our results offer supporting evidence.

I. Regression Tests: Theory

Treasury bonds with maturities longer than a year are not issued on a regular basis, and only irregularly spaced maturities are available. To estimate a term structure for regularly spaced maturities, some method of interpolation must be used. We use such a method (see the Appendix) to construct end-of-month prices for 1- to 5-year discount bonds. From the prices, we calculate forward rates, returns, and interest rates for annual maturities to 5 years.

The tests of the information in forward rates about current expected returns and future interest rates are simple regressions of future returns and changes in interest rates on current forward rates. As in most term structure work, however, even simple tests require a tedious notation.

A. *Definitions of Variables*

The return on an x-year discount bond bought at time t and sold at $t+x-y$, when it has y years to maturity, is defined as

$$(1)\quad h(x, y: t+x-y) = \ln p(y: t+x-y) - \ln p(x:t),$$

where ln indicates a natural log, and $p(x:t)$ is the price of the bond at t. Symbols before a colon are the maturities that define a variable. The symbol after the colon is the time the variable is observed. Since most of the empirical variables are annual, time is measured in annual increments. For example, $h(5,4:t+1)$ is the 1-year return from t to $t+1$ on a 5-year bond.

The yield $r(x:t)$ on a discount bond with \$1 face value and x years to maturity at t is defined as

$$(2)\qquad r(x:t) = -\ln p(x:t).$$

The yield $r(1:t)$ on a 1-year bond is called the 1-year spot rate. It has a prominent role in the tests.

The time t 1-year forward rate for the year from $t+x-1$ to $t+x$ is

$$(3)\quad f(x, x-1:t) = \ln p(x-1:t) - \ln p(x:t) = r(x:t) - r(x-1:t).$$

For example, $f(5,4:t)$ is the forward rate for the year from $t+4$ to $t+5$.

The time t price of an x-year discount bond that pays \$1 at maturity is the present value of the \$1 payoff discounted at the time t expected values (E_t) of the future 1-year returns on the bond,

$$(4)\quad p(x:t) = \exp[-E_t h(x, x-1:t+1) - E_t h(x-1, x-2:t+2) - \dots - E_t r(1:t+x-1)].$$

Equation (4) is a tautology, implied by the definition of returns. It acquires testable content when we add the hypothesis that the expected returns in (4) are rational forecasts used by the market to set $p(x:t)$. Equation (4) then says that the price contains rational forecasts of equilibrium expected returns. This hypothesis about the price is the basis of the tests.

B. *Forward Rates and Future Spot Rates*

For example, the forward rate $f(x, x-1:t)$ can be viewed as the rate set at t on a contract to purchase a 1-year bond at $t+x-1$. Motivated by this view, the literature has long been concerned with the hypothesis that the forward rate rationally forecasts the 1-year spot rate, $r(1:t+x-1)$, to be observed at $t+x-1$. To focus on the forecast of the spot rate in $f(x, x-1:t)$, we sum the first $x-1$ expected returns in (4) and write the price as

$$(5)\quad p(x:t) = \exp[-E_t h(x, 1:t+x-1) - E_t r(1:t+x-1)].$$

Substituting (5) into (3) and subtracting the 1-year spot rate $r(1:t)$ gives

$$(6)\quad f(x,x-1:t)-r(1:t) = \left[E_t r(1:t+x-1)-r(1:t)\right] + \left[E_t h(x,1:t+x-1)-r(x-1:t)\right].$$

We call $f(x,x-1:t)-r(1:t)$ the forward-spot spread. Our tests of the information in the forward rate $f(x,x-1:t)$ about the future spot rate $r(1:t+x-1)$ then center on the slope in the forecasting regression,

$$(7)\quad r(1:t+x-1)-r(1:t) = a_1 + b_1\left[f(x,x-1:t)-r(1:t)\right] + u_1(t+x-1).$$

Evidence that b_1 is greater than 0.0 implies that the forward-spot spread observed at time t has power to forecast the change in the 1-year spot rate $x-1$ years ahead.

Equation (6) holds for realized returns as well as expected values,

$$(8)\quad f(x,x-1:t)-r(1:t) = \left[r(1:t+x-1)-r(1:t)\right] + \left[h(x,1:t+x-1)-r(x-1:t)\right].$$

It follows that the regression (7) is complementary to the regression

$$(9)\quad h(x,1:t+x-1)-r(x-1:t) = -a_1 + (1-b_1)\left[f(x,x-1:t)-r(1:t)\right] - u_1(t+x-1).$$

As indicated, the intercepts in (7) and (9) sum to 0.0, the residuals sum to 0.0 every period, and, most interesting, the slopes sum to 1.0. Thus, the slope in (7) estimates the split of variation in the forward-spot spread between the two terms of (6): (*i*) the forecasted change in the 1-year spot rate from t to $t+x-1$; and (*ii*) the premium of the $(x-1)$-year expected return on an x-year bond over the time t yield on an $(x-1)$-year bond.

Since b_1 is a constant, the estimated split of variation in the forward-spot spread does not change through time. This means that (7) can tell us that the forward-spot spread has power to forecast the change in the spot rate; but the regression fitted values only track all variation in the forecasts when the expected change in the spot rate and the expected premium in (6) always vary in fixed proportion. This is a limitation of (7) and of similar regressions outlined below.

Expression (6) for the forward-spot spread combines expected returns in (4) to focus on information in the time t price, $p(x:t)$, about the return, $h(x,1:t+x-1)$, and the spot rate, $r(1:t+x-1)$, to be observed at the beginning of the last year in the life of the x-year bond. We turn now to a different grouping of the expected returns in (4) which focuses on the information in $p(x:t)$ about the 1-year return, $h(x,x-1:t+1)$, and the yield, $r(x-1:t+1)$, to be observed in 1 year.

C. *Forward Rates and* 1-*Year Expected Returns*

If we sum the last $x-1$ expected returns in (4), the price of an x-year bond is

$$(10)\quad p(x:t)=\exp\left[-E_t h(x,x-1:t+1) - E_t r(x-1:t+1)\right].$$

Substituting (10) into (3) and subtracting the spot rate $r(1:t)$ gives

$$(11)\quad f(x,x-1:t)-r(1:t) = \left[E_t h(x,x-1:t+1)-r(1:t)\right] + \left[E_t r(x-1:t+1)-r(x-1:t)\right].$$

Thus, when (10) is used for the price of an x-year bond, the forward-spot spread contains $E_t(x,x-1:t+1)-r(1:t)$, the time t expected premium of the 1-year return on an x-year bond over the 1-year spot rate. But the forward-spot spread also contains the

TABLE 1—TERM PREMIUM REGRESSIONS: ESTIMATES OF (12): 1964–85
$h(x, x-1:t+1) - r(1:t) = a_2 + b_2[f(x, x-1:t) - r(1:t)] + u_2(t+1)$

						Residual Autos (Yearly Lag)				
Dependent	a_2	$s(a)$	b_2	$s(b)$	R^2	1	2	3	4	5
$h(2,1:t+1)-r(1:t)$	−.21	.41	.91	.28	.14	−.01	−.12	−.07	−.17	−.01
$h(3,2:t+1)-r(1:t)$	−.51	.68	1.13	.37	.11	−.18	−.12	.03	−.17	−.05
$h(4,3:t+1)-r(1:t)$	−.91	.92	1.42	.45	.11	−.23	−.10	.02	−.14	−.08
$h(5,4:t+1)-r(1:t)$	−1.06	1.31	.93	.53	.05	−.17	−.11	.03	−.17	−.10

Note: $r(1:t)$ is the 1-year spot rate observed at t; $h(x, x-1:t+1)$ is the 1-year return (t to $t+1$) on an x-year bond, and $h(x, x-1:t+1) - r(1:t)$ is the term premium in the 1-year return. $f(x, x-1:t)$ is a 1-year forward rate observed at t, and $f(x, x-1:t) - r(1:t)$ is the forward-spot spread. The regression estimates the expected value of the term premium to be observed at $t+1$, conditional on the forward-spot spread observed at t. The standard errors, $s(a)$ and $s(b)$, of the regression coefficients are adjusted for possible heteroscedasticity and for the autocorrelation induced by the overlap of monthly observations on annual returns. (See Halbert White, 1980, and Lars Peter Hansen, 1982.) The regression R^2 is adjusted for degrees of freedom. The sample size is 252, corresponding to the period January 1964 to December 1984 for the forward-spot spreads and January 1965 to December 1985 for the term premiums. If the true autocorrelations are 0.0, the standard error of the estimated residual autocorrelations is about 0.065. The data are derived from the U.S. Government Bond File of the Center for Research in Security Prices (CRSP) of the University of Chicago. See the Appendix.

expected change from t to $t+1$ in the yield on $(x-1)$-year bonds. If yields are random walks, the expected yield change in (10) is 0.0, and $f(x, x-1:t) - r(1:t)$ is $E_t h(x, x-1:t+1) - r(1:t)$. The common finding that forward rates have little power to forecast interest rates suggests a world where yields are close to random walks.

We call $h(x, x-1:t+1) - r(1:t)$ the term premium in the 1-year return on an x-year bond. Our tests of the information in time t forward rates about time t 1-year expected returns then center on the slope in the regression,

$$(12)\quad h(x, x-1:t+1) - r(1:t) = a_2 + b_2[f(x, x-1:t) - r(1:t)] + u_2(t+1).$$

Evidence that b_2 is positive implies that the expected value of the term premium varies through time. Moreover, like (6), (11) holds for the realized returns, $h(x, x-1:t+1)$ and $r(x-1:t+1)$, as well as for their expected values. It follows that $1-b_2$ in (12) is the slope in the complementary regression,

$$(13)\quad r(x-1:t+1) - r(x-1:t) = -a_2 + (1-b_2)[f(x, x-1:t) - r(1:t)] - u_2(t+1).$$

Evidence that b_2 differs from 1.0 means that the forward-spot spread forecasts the change in the $(x-1)$-year yield 1 year ahead.

In short, the slope in (12) splits variation in the forward-spot spread between the 1-year expected term premium and expected yield change in (11), just as the slope in (7) splits variation in the forward-spot spread between the two multiyear forecasts in (6).

II. Expected Term Premiums

A. *Time-Varying Expected Term Premiums*

Estimates of the term-premium regression (12) are in Table 1. Three slopes are more than 3.0 standard errors from 0.0, and the fourth is 1.75 standard errors from 0.0. We infer that expected term premiums in 1-year returns for maturities to 5 years vary through time, and so are typically nonzero. The results are in contrast to previous work of Kessel, McCulloch, Fama (1984b), and others that finds no convincing evidence of incremental expected returns for maturities beyond a year. The tests extend to longer maturities the conclusion of Richard Startz (1982), Shiller et al., and Fama (1984a, 1986), that expected bill returns contain time-varying maturity premiums. The evidence also confirms Shiller's (1979) claim that if bond prices are rational, the high variability of

TABLE 2—AUTOCORRELATIONS, MEANS, AND STANDARD DEVIATIONS: 1964-85

Variable	$\bar{x}$	$s(x)$	Autocorrelations (Monthly Lag) 1	2	3	4	5	6	12	24	36	48	60
Spot and Forward Rates: $r(1:t)$ and $f(x,x-1:t)$													
$r(1:t)$	7.49	2.90	.97	.93	.89	.86	.84	.81	.71	.48	.31	.15	.09
$f(2,1:t)$	7.59	2.81	.97	.94	.92	.89	.87	.86	.75	.59	.43	.24	.12
$f(3,2:t)$	7.74	2.69	.96	.94	.92	.90	.89	.87	.75	.59	.41	.23	.11
$f(4,3;t)$	7.80	2.79	.96	.94	.92	.91	.89	.87	.75	.59	.44	.24	.11
$f(5,4:t)$	7.74	2.65	.93	.90	.89	.88	.87	.85	.75	.60	.41	.26	.12
Term Premiums: $h(x,x-1:t+1)-r(1:t)$													
$h(2,1:t+1)-r(1:t)$	−.11	2.18	.90	.80	.72	.65	.59	.53	.08	−.10	−.16	−.24	−.05
$h(3,2:t+1)-r(1:t)$	−.23	3.75	.89	.79	.70	.63	.56	.49	−.01	−.09	−.11	−.22	−.09
$h(4,3:t+1)-r(1:t)$	−.46	5.13	.89	.80	.71	.64	.57	.48	−.04	−.08	−.09	−.21	−.10
$h(5,4:t+1)-r(1:t)$	−.83	6.36	.89	.80	.73	.66	.58	.49	−.04	−.07	−.08	−.22	−.13
Forward-spot spreads: $f(x,x-1:t)-r(1:t)$													
$f(2,1:t)-r(1:t)$	.10	.91	.82	.78	.71	.64	.59	.53	.34	−.12	−.42	−.22	.03
$f(3,2:t)-r(1:t)$	.26	1.13	.78	.66	.57	.51	.48	.45	.29	−.14	−.39	−.23	.01
$f(4,3:t)-r(1:t)$	.31	1.22	.74	.66	.62	.53	.51	.48	.33	−.10	−.39	−.25	−.03
$f(5,4:t)-r(1:t)$	.25	1.54	.77	.62	.53	.52	.53	.47	.32	−.14	−.34	−.23	−.09
Residuals from Spot Rate Autoregression (14)													
$e(t)$	.00	.69	.15	−.08	−.12	−.11	.08	−.08	−.07	.06	−.07	−.02	−.04

Note: $r(1:t)$ is the 1-year spot rate observed at t; $h(x,x-1:t+1)$ is the 1-year return (t to $t+1$) on an x-year bond; $f(x,x-1:t)$ is a 1-year forward rate observed at t; $e(t)$ is the residual from the first-order autoregression (14) fit to the monthly time series of $r(1:t)$. The sample size is 252, corresponding to the period January 1964 to December 1984 for the forward-spot spreads and the period January 1965 to December 1985 for the term premiums in 1-year returns. If the true autocorrelations are 0.0, the standard error of the estimated autocorrelations is about 0.065. The data are derived from the CRSP U.S. Government Bond File. See the Appendix.

yields on longer-term bonds implies time-varying expected returns.

The slopes b_2 in the term-premium regressions range from 0.91 to 1.42. All are within one standard error of 1.0. We can infer that the slopes (equal to $1-b_2$ and with the same standard errors as b_2) in the complementary yield-change regression (13) are within one standard error of 0.0. The results suggest that when forward-spot spreads are viewed as in (11), variation in current spreads is mostly variation in the term premiums in current 1-year expected returns, and forward-spot spreads do not predict yield changes 1 year ahead. The evidence extends to longer maturities Fama's (1984a, 1986) conclusion for bills that forward rates are close to current expected returns.

B. *The Behavior of Expected Term Premiums*

Kessel, McCulloch, Fama (1984b), and others show that inferences from average returns about average expected returns on longer-maturity bonds are imprecise because of the high variability of returns. An alternative approach is to use the term-premium regressions as a license to infer average expected returns from average forward rates. The average forward rates in Table 2 show no strong tendency to increase or decrease across longer maturities. The average value of the 4- to 5-year forward rate, $f(5,4:t)$, only exceeds the average value of the 1-year spot rate, $r(1:t)$, by 0.25 percent per year.

However, the structure of forward rates varies through time, and the picture provided by average forward rates is misleading. Figure 1 plots the 5-year forward-spot spread. General patterns of variation are similar for other maturities. If forward-spot spreads are expected term premiums in 1-year returns, Figure 1 shows the general path of the variation through time of expected term premiums. The forward-spot spread is characterized by alternating runs of positive and negative values. At least after 1970, there seems to be a relation between the sign of

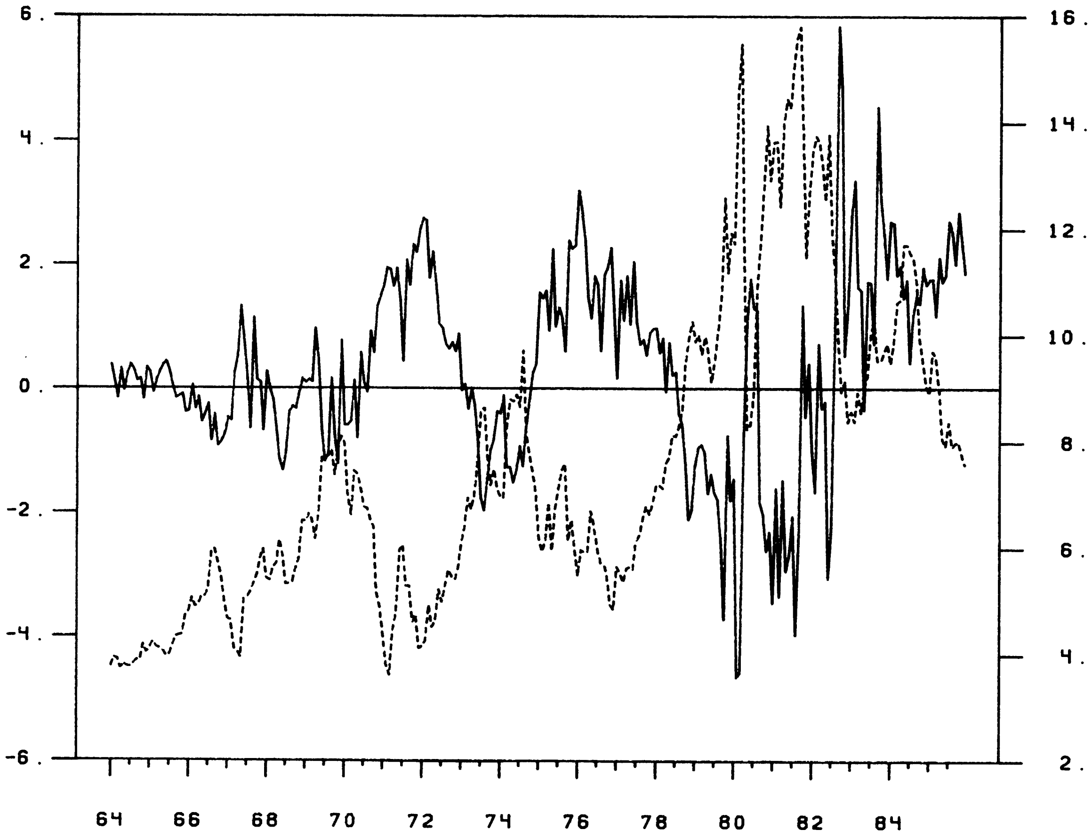

FIGURE 1. SPOT RATE AND 5-YEAR FORWARD-SPOT SPREAD

Note: 5-year forward-spot spread, $f(5,4:t)-r(1:t)$, (solid line) and the 1-year spot rate, $r(1:t)$, (dashed line). The left vertical axis is the scale (percent per year) for the spread, and the right is the scale for the spot rate. The horizontal axis is the date t.

the forward-spot spread and the business cycle. Positive forward-spot spreads in Figure 1 tend to be associated with periods of strong business activity, for example, 1970–72, 1975–78, 1983–85. Negative spreads tend to occur during the recessions of 1973–74 and 1979–82.

In short, the average forward rates in Table 2 suggest that the term structure of expected 1-year returns on 1- to 5-year Treasury bonds is on average flat. But Figure 1 shows that a flat term structure of forward rates is not typical. The path of the forward-spot spread in Figure 1 suggests that expected term premiums are typically nonzero and vary between positive and negative values. Such changes in the ordering of expected returns, and their apparent relation to the business cycle, pose an interesting challenge to term structure models that can accommodate time-varying expected returns.

C. *The Effects of Measurement Error*

A caveat about the term-premium regressions is in order. The spot rate $r(1:t)$ is obtained from a 1-year bill, but the implied prices of discount bonds used to estimate forward rates and returns for longer maturities involve interpolation that can produce measurement error. Errors in the long-maturity price $p(x:t)$ in $f(x,x-1:t)$ tend to bias the slope in the term-premium regression (12) toward 1.0, since $p(x:t)$ is the

TABLE 3—REGRESSION FORECASTS OF THE CHANGE IN THE SPOT RATE

								Residual Autos (Yearly Lag)				
Dependent	a	$s(a)$	b_1	$s(b_1)$	b_2	$s(b_2)$	R^2	1	2	3	4	5
$\boldsymbol{r(1:t+x-1)-r(1:t)=a+b_1[f(x,x-1;t)-r(1:t)]+u(t+x-1)}$												
$r(1:t+1)-r(1:t)$	.21	.41	.09	.28			.00	−.11	−.12	−.07	−.17	−.01
$(1:t+2)-r(1:t)$	.40	.73	.69	.26			.08	.39	−.21	−.36	−.33	−.08
$r(1:t+3)-r(1:t)$	.57	.75	1.30	.10			.24	.52	.04	−.40	−.46	−.25
$r(1:t+4)-r(1:t)$	1.12	.61	1.61	.34			.48	.38	.09	−.09	−.23	−.27
$\boldsymbol{r(1:t+x-1)-r(1:t)=a+b_2[[\hat{r}(1:t+x-1)-r(1:t)]+u(t+x-1)}$												
$r(1:t+1)-r(1:t)$	.03	.44			.87	.40	.16	−.01	−.05	−.02	−.12	.03
$r(1:t+2)-r(1:t)$	.16	.70			.88	.20	.26	.43	−.08	−.15	−.10	.04
$r(1:t+3)-r(1:t)$	.22	.84			.90	.13	.33	.57	.16	−.15	−.14	−.04
$r(1:t+4)-r(1:t)$	.37	.80			.91	.21	.36	.58	.29	.03	−.18	−.10
$\boldsymbol{r(1:t+x-1)-r(1:t)=a+b_1[f(x,x-1:t)-r(1:t)]+b_2[\hat{r}(1:t+x-1)-r(1:t)]+u(t+x-1)}$												
$r(1:t+1)-r(1:t)$	.04	.43	−.14	.24	.90	.39	.16	−.03	−.04	−.00	−.09	.05
$r(1:t+2)-r(1:t)$	.15	.74	.19	.30	.82	.17	.26	.44	−.07	−.17	−.15	.01
$r(1:t+3)-r(1:t)$	.22	.84	.76	.43	.69	.22	.40	.56	.19	−.15	−.27	−.17
$r(1:t+4)-r(1:t)$	.79	.79	1.21	.45	.38	.20	.51	.47	.19	.03	−.14	−.23

Note: $r(1:t)$ is the 1-year spot rate observed at t; $f(x,x-1:t)$ is a 1-year forward rate observed at t; $\hat{r}(1:t+x-1)$ is the time t forecast of $r(1:t+x-1)$ from the first-order autoregression (14) fit to the time-series of $r(1:t)$. The standard errors of the regression coefficients are adjusted for possible heteroscedasticity and for the autocorrelation induced by the overlap of monthly observations on annual or multiyear changes in the spot rate. See White and Hansen. The sample size in the regressions for the 1-year change in the spot rate, $r(1:t+1)-r(1:t)$ is 252, corresponding to the period January 1964 to December 1984 for the forward-spot spread and the period January 1965 to December 1985 for the 1-year changes in the 1-year spot rate. An additional year (12 months) of data is lost each time the forecast horizon $t+x-1$ is extended an additional year. Thus, the regressions for the 4-year change, $r(1:t+4)-r(1:t)$, have 216 observations. Under the hypothesis that the true autocorrelations are 0.0, the standard error of the estimated residual autocorrelations is about 0.065. The data are derived from the CRSP U.S. Government Bond File. See the Appendix.

purchase price for the return $h(x,x-1:t+1)$. Errors in the short-maturity price $p(x-1:t)$ in $f(x,x-1:t)$ tend to bias the slope in (12) toward 0.0 since $-\ln p(x-1:t)$ is the yield $r(x-1:t)$ in the complementary yield change regression (13). The net effect of measurement error on the slopes in the term-premium regressions is thus difficult to predict.

It is easier to predict the effect of measurement error on the spot-rate forecasting regression (7). The future spot rate $r(1:t+x-1)$ is calculated from the price of a 1-year bill at $t+x-1$. Measurement errors in the time t prices in $f(x,x-1:t)$ tend to bias the slope in (7) toward 0.0 and so attenuate our ability to identify forecast power in the forward rate. The forecast power we find in the tests that follow is thus in spite of any bias due to measurement error in forward rates.

III. Forecasts of 1-Year Spot Rates

A. *Estimates of the Spot-Rate Forecasting Regression*

Since slopes close to 1.0 in the estimates of the term-premium regression (12) suggest that forward rates do not forecast yields 1 year ahead, intuition suggests that they will not forecast longer-term changes in rates. Intuition is not confirmed by the slope estimates for regression (7) in Table 3. The slopes are more than 2.6 standard errors from 0.0 for all forecasts beyond a year. The forward-spot spread, $f(x,x-1:t)-r(1:t)$, forecasts the change in the 1-year spot rate, $r(1:t+x-1)-r(1:t)$, 2 to 4 years ahead. Moreover, forecast power improves with the forecast horizon: $f(3,2:t)-r(1:t)$ explains 8 percent of the variance of the change in the spot rate 2 years ahead; $f(4,3:t)-$

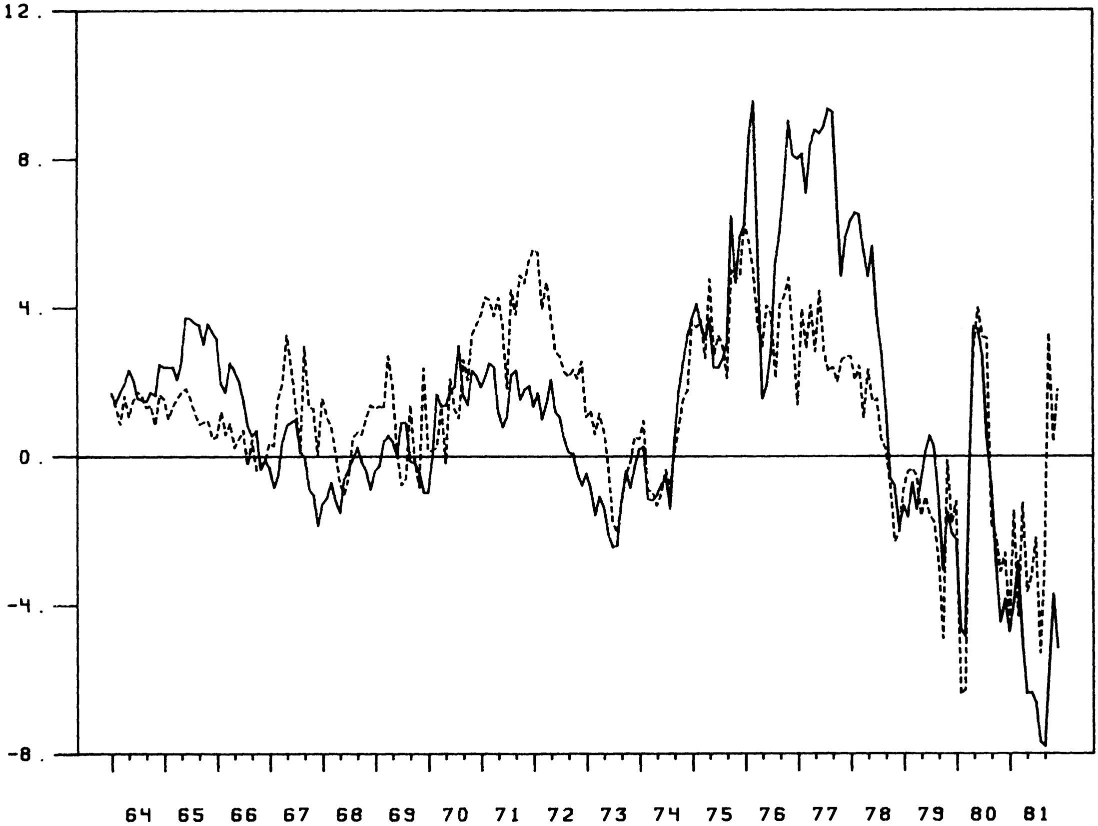

FIGURE 2. 4-YEAR CHANGE IN SPOT RATE AND FORECAST FROM REGRESSION (7)

Note: 4-year change in the spot rate, $r(1:t+4)-r(1:t)$, (solid line) and the forecasted change (dashed line) from the regression (7) of $r(1:t+4)-r(1:t)$ on the forward-spot spread $f(5,4:t)-r(1:t)$. The vertical axis is percent per year and the horizontal axis is t, the date of the forecast.

$r(1:t)$ explains 24 percent of the variance of the 3-year change; $f(5,4:t)-r(1:t)$ explains 48 percent of the variance of the 4-year change.

Figure 2 plots 4-year changes in the 1-year spot rate and the fitted values from the regression of $r(1:t+4)-r(1:t)$ on $f(5,4:t)-r(1:t)$. The figure suggests that the high R^2 (0.48) of the regression reflects consistent long-term forecast power during the sample period. We argue that long-term forecast power is largely due to slow mean reversion of the spot rate.

B. *Forecast Power and Mean Reversion*

The autocorrelations of $r(1:t)$ in Table 2 are close to 1.0 at short lags, but they decay across longer lags. The pattern suggests that month-to-month levels of the 1-year spot rate are highly autocorrelated, but the spot rate has a slow mean-reverting tendency.

The Appendix shows that if the 1-year spot rate is mean-reverting (stationary), then for long forecast horizons the expected change in the spot rate due to mean reversion explains half the variance of the actual change. If the spot rate is slowly mean reverting, the expected change explains more of the variance of the change for longer forecast horizons. Recall that the proportions of the variance of the change in the 1-year spot rate explained by forward-spot spreads in the estimates of (7) increase with the forecast horizon and reach 0.48 for 4-year changes. Thus, the forecast power of forward-spot spreads conforms to what is predicted by slow mean reversion.

It is also easy to show that if the spot rate is highly autocorrelated but slowly mean reverting, the correlation of the nonoverlapping changes, $r(1:t+T)-r(1:t)$ and $r(1:t)-r(1:t-T)$, is close to 0.0 for small values of T but approaches -0.5 for large values of T. The correlations between nonoverlapping 1-, 2-, 3-, and 4-year changes in the 1-year spot rate are -0.13, -0.29, -0.51, and -0.52.

B. *Forecast Power and Mean Reversion: Direct Evidence*

Although slow mean reversion of the spot rate is sufficient to explain the long-term forecast power of forward rates, the pattern of decay of the autocorrelations of the spot rate suggests that a first-order autoregression (AR1) is a reasonable model for the 1964–85 period. Fitting an AR1 to monthly observations on the 1-year spot rate yields

$$(14) \quad r(1:t) = \underset{(.121)}{.257} + \underset{(.015)}{.968} r(1:t-1) + e(t),$$

where the numbers in parentheses are standard errors. The autocorrelations of the residuals (Table 2) are generally close to 0.0. The first-order residual autocorrelation, 0.15, suggests that the model can be improved by adding a (1-month) moving-average term. Since we are interested in forecasts of spot-rate changes over long horizons, and since our goal is to document a mean-reverting tendency in a simple way, we stick with the AR1.

We use (14), fit to monthly levels of the 1-year spot rate, to forecast changes 1 to 4 years ahead. Regressions of actual on forecasted changes test for forecast power due to slow mean reversion. If the spot rate has a slow mean-reverting tendency which is well-approximated by the estimated AR1, the regressions of changes on the changes forecast by the AR1 will have forecast power that increases with the forecast horizon. Note, however, that the estimated AR1 slope, 0.968, is barely 2.0 standard errors from 1.0. If the spot rate is actually a random walk, the regressions of changes on the changes forecast by the estimated AR1 will have no power.

Regression intercepts (Table 3) that are all within 1.0 standard error of 0.0 and slopes within 1.0 standard error of 1.0 suggest that the AR1 forecasts of spot-rate changes are unbiased. As predicted by slow mean reversion, the forecasts explain more of the variance of spot-rate changes for longer forecast horizons. Explained variance rises from 16 percent for 1-year changes to 36 percent for 4-year changes. However, variance explained never reaches 50 percent, the limit predicted by mean reversion. An AR1 may not be the best model for the spot rate. But the AR1 serves its purpose. Its power to forecast spot-rate changes is consistent with a tendency toward mean reversion of the spot rate.

C. *Forward Rate vs. AR1 Forecasts*

Is the forecast power of forward rates solely due to mean reversion of the spot rate? Multiple regressions (Table 3) of spot-rate changes on forward-spot spreads and the AR1 forecasts are a test. In the regressions for 1- and 2-year changes, the slopes for the forward-spot spread are close to 0.0, while the slopes for the AR1 forecasts are close to 1.0 and more than 2.0 standard errors from 0.0. In the multiple regressions for 3- and 4-year changes, both the forward-spot spreads and the AR1 forecasts have marginal explanatory power (slopes almost or more than 2.0 standard errors from 0.0).

The fact that the AR1 dominates forward-spot spreads in the 1- and 2-year regressions does not necessarily mean that the market misses some of the mean reversion of the spot rate. We know from (6) that $f(x,x-1:t)-r(1:t)$ contains both the spot-rate forecast, $E_t r(1:t+x-1)-r(1:t)$, and the premium, $E_t h(x,1:t+x-1)-r(x-1:t)$. If the expected change in the spot rate is a varying proportion of the forward-spot spread, the fixed-coefficient regression of $r(1:t+x-1)-r(1:t)$ on $f(x,x-1:t)-r(1:t)$ will not track all the information in the forward rate about the future spot rate.

There is evidence that variation in forward rates is not due entirely to forecasts of a mean-reverting spot rate. Although forward-spot spreads move opposite the spot rate, the correlations of the spot rate with 2- to 5-year forward-spot spreads (-0.25, -0.37, -0.30, and -0.42) are far from -1.0. If forward rates were driven only by mean reversion of the spot rate, our longest maturity forward rate, $f(5,4:t)$, would be much less variable than the spot rate. Table 2 shows that the standard deviation of $f(5,4:t)$ is within 10 percent of that of $r(1:t)$.

On the other hand, the fact that forward-spot spreads have marginal explanatory power relative to the AR1 in 3- and 4-year regression forecasts may reflect deficiencies of the AR1. One possibility is that reversion to a constant mean is not a complete story for the spot rate. Such mean reversion predicts that forward-spot spreads are positive (the spot rate is expected to increase) when the spot rate is low and negative when the spot rate is high. The plots of the spot rate and the 5-year forward-spot spread in Figure 1 confirm this prediction for most of the sample period. However, some of the lowest values of the spot rate occur in 1964–68. Mean reversion predicts positive expected changes in spot rates for this period, but forward-spot spreads are not systematically positive. Likewise, when the spot rate drops in 1982, it remains above its sample mean. Simple mean reversion predicts that forward-spot spreads should not become positive, but they do.

Scott Ulman and John Wood (1983) suggest that the mean of the spot rate rises with the end of the gold standard in 1971 because a fiduciary currency means higher average inflation rates. A rise in the mean of the spot rate is consistent with the behavior of the spot rate and the forward-spot spread in Figure 1. John Campbell and Shiller (1984) suggest a time-series model for the spot rate that includes mean reversion, but toward a mean that can change through time.

The important point for our purposes is that the Table 3 regressions are sufficient to conclude that much of the forecast power of forward rates is due to a mean-reverting tendency of the spot rate. But we cannot infer on the basis of our simple tests that the forecast power of forward rates is better or worse than expected on the basis of the mean reversion of the spot rate.

Finally, the autocorrelations of forward rates (Table 2) are close to those of the spot rate. The same slow decay is observed in autocorrelations of 1- to 5-year yields (not shown). A slow mean-reverting tendency is apparently a general property of interest rates—an appealing conclusion with strong roots in term structure theory. However, because the autocorrelations of interest rates are close to 1.0 for short lags, recent empirical studies (for example, Nelson-Plosser and Fama-Gibbons) tend to conclude that interest rates are approximately random walks. Our evidence that long-term changes in rates are predictable suggests that the slow decay of the autocorrelations should instead be emphasized.

V. Conclusions

A. *Forward Rates and the Term Structure of Expected Returns*

The estimates of the term-premium regression (12) allow us to infer that 1-year expected returns for U.S. Treasury maturities to 5 years, measured net of the interest rate on a 1-year bond, vary through time. Moreover, at least during the 1964–85 period, this variation of expected term premiums seems to be related to the business cycle. Expected term premiums are mostly positive during good times but mostly negative during recessions.

Differences in expected returns are usually interpreted as rewards for risk. Our evidence, like that for shorter maturities in Fama (1986), suggests that the ordering of risks and rewards changes with the business cycle. This behavior of expected returns is inconsistent with simple term structure models, like the liquidity preference hypothesis of Hicks and Kessel in which expected returns always increase with maturity. Perhaps it can be explained by models, like those of Merton, Long, Breeden, and Cox et al., that allow time-varying expected returns. The challenge is apparent.

B. *Forward Rates and Future Spot Rates*

Like earlier work, we find little evidence that forward rates can forecast near-term changes in interest rates. When the forecast horizon is extended, however, forecast power improves, and 1-year forward rates forecast changes in the 1-year spot rate 2 to 4 years ahead. We conclude that this forecast power reflects a slow mean-reverting tendency of interest rates.

Like any interest rate, the 1-year spot rate on Treasury bonds can be split into an expected real return and an expected inflation rate. The mean reversion of the spot rate likely implies that both of its components are mean reverting, a hypothesis with strong economic appeal. However, theory does not suggest that the processes that generate expected inflation and expected real returns should be the same, or that the processes should not change through time. We interpret the mean reversion of interest rates documented here as a tendency with details to be documented by future work.

APPENDIX

A. *Data*

The U.S. Government Bond File of the Center for Research in Security Prices (CRSP) has end-of-month data for all U.S. Treasury securities. We use the data to estimate end-of-month term structures for taxable, noncallable bonds for annual maturities to 5 years. The approach is outlined below. Our data are available to subscribers to the CRSP bond file.

Each month a term structure of 1-day continuously compounded forward rates is first calculated from available maturities. Bills are used for maturities to a year. To extend beyond a year, the pricing assumption is that the daily forward rate for the interval between successive maturities is the relevant discount rate for each day in the interval. Suppose daily forward rates for month t are calculated for maturities to T and the next bond matures at $T+k$. Coupons on the bond to be received prior to T are priced with the daily forward rates from t to each payment date. Coupons and the principal to be received after T are priced with the daily forward rates from t to T and with the (solved for) daily forward rate for T to $T+k$ that equates the price of the bond at t to the value of all payments. These calculations generate a step-function term structure in which 1-day forward rates are the same between successive maturities.

Prior to the recent large deficits, the number of noncallable, fully taxable bonds available each month is small. From 1964 onward there is at least one bond in each 1-year maturity interval to 5 years, but often there are few bonds beyond 5 years. We sum the daily forward rates to generate end-of-month term structures of yields for annual maturities to 5 years. The yields are used to calculate implied prices of 1- to 5-year discount bonds, from which we calculate the term-structure variables used in the tests.

B. *Forecast Power and Mean Reversion*

We use a first-order autoregression (AR1) to illustrate that predictable changes in the spot rate due to slow mean reversion are more apparent over longer horizons. Suppose $z(t)$ is an AR1 with parameter ϕ and mean μ,

$$z(t)=\delta+\phi z(t-1)+\in(t), \qquad \mu=\delta/(1-\phi). \tag{A1}$$

The time t expected value of $z(t+T)$ is (see Nelson, 1973, p. 148)

$$E_t z(t+T)=\mu+\phi^T[z(t)-\mu], \tag{A2}$$

and the expected change in $z(t)$ from t to $t+T$ is

$$E_t z(t+T)-z(t) = [z(t)-\mu](\phi^T-1). \tag{A3}$$

If ϕ is close to 1.0, the expected change in $z(t)$ is small for small values of T. The expected change in $z(t)$ increases with the forecast horizon and approaches $\mu - z(t)$: $z(t)$ is expected to revert to its mean μ.

The variance of the expected change in $z(t)$ is

$$\sigma^2[E_t z(t+T) - z(t)] = \sigma^2(z)(\phi^T - 1)^2, \tag{A4}$$

which grows with T and approaches $\sigma^2(z)$. How much of the variance of the T-period change is explained by the expected change? Since $\text{cov}[z(t+T), z(t)]$ approaches 0.0 for large values of T, the variance of $z(t+T) - z(t)$ approaches $2\sigma^2(z)$. Since $\sigma^2[E_t z(t+T) - z(t)]$ approaches $\sigma^2(z)$, for long forecast horizons the expected change in $z(t)$ due to the mean reversion of $z(t)$ explains half the variance of $z(t+T) - z(t)$. This result is a general property of stationary processes; it is not special to an AR1.

For an AR1 with ϕ close to 1.0, the ratio of the variance of the expected change to the variance of the change approaches 0.5 from below. For example, if $\phi = 0.95$, $\sigma^2[E_t z(t+1) - z(t)]$ is $0.025\sigma^2[z(t+1) - z(t)]$. Thus, the expected change explains more of the variance of the change for longer horizons. This is a rather general implication of slow mean reversion.

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