## Notes on Taxes and Growth

Most taxes are levied on consumption, on labor income, or on the income from capital. The neoclassical model of economic growth is the appropriate tool for analyzing a tax on income from capital. We begin with the top half of Figure 1, which is similar to Mankiw's Figure 4-12 except that it shows the production function, $f(k)$, rather than $s f(k)$.

First, let's examine the case with no capital income tax by reviewing some results we have seen before. We know that the slope of the production function is simply the MPK. We also know that steady-state consumption (per unit of effective labor) is maximized at the capital stock which equates $\delta+n+g$ with the slope of the production function. This condition can be written as

$$
M P K=\delta+n+g
$$

or

$$
M P K-\delta=n+g .
$$

Remembering that $M P K-\delta$, the net marginal product of capital, is equal to the interest rate, we see that the consumption-maximizing capital stock is the one that equates $n+g$ with the interest rate. In Figure 1, this capital stock is $k^{*}$. The equality between $n+g$ and the net marginal product of capital when capital is equal to $k^{*}$ is shown in the bottom half of Figure 1.

We now state a new result without proving it. Suppose that an individual's lifetime utility is given by the sum of the discounted utility of consumption in each period of life

$$
U=u\left(C_{1}\right)+\frac{u\left(C_{2}\right)}{1+\rho}+\frac{u\left(C_{3}\right)}{(1+\rho)^{2}}+\ldots
$$

where $C_{t}$ is consumption in period $t$. The subjective discount rate $\rho$ is called the rate of time preference. If $\rho>$ 0 , people place less weight on future utility than on current utility. The rate of time preference is similar to an interest rate and measures how heavily people discount the future relative to the present. For example, if $\rho$ is 0.02 , people discount future utility at the subjective rate of two percent per period. Our new result states that, if people have utility functions like the one above, capital will be accumulated up to the point where the steadystate interest rate equals $\rho$. This capital stock (per effective labor unit) is labeled $k_{0}$ in Figure 1. It is determined by the intersection of the net MPK curve and $\rho$. As drawn, the steady-state capital stock $k_{0}$ is less than the consumption-maximizing capital stock $k^{*}$. Most economists think this is a reasonable assumption.

We now examine the effects of introducing a tax on income from capital. We assume that the net

[^0]marginal product of capital is taxed at a rate $\tau_{k}$. Interest income is also subject to this tax, and interest payments are deductible from taxable income. In the absence of taxes, firms invest up to the point where the net marginal product of capital equals the interest rate. With the tax structure just described, firms find it optimal to invest up to the point where the after-tax interest rate equals the after-tax net marginal product of capital:
$$
r\left(1-\tau_{k}\right)=(M P K-\delta)\left(1-\tau_{k}\right) .
$$

At first glance, it may appear that the tax has no effect on the firm's investment decision, since the tax factor cancels out of both sides of this equation. This conclusion is wrong, however, because the steady-state after-tax interest rate must equal $\rho$ :

$$
\rho=r\left(1-\tau_{k}\right)=(M P K-\delta)\left(1-\tau_{k}\right)
$$

The higher the capital income tax rate, the higher the before-tax interest rate (and the before-tax net MPK) must be if this equation is to be satisfied. A higher before-tax net MPK implies a lower capital stock.

These results are shown in the bottom half of Figure 1. The after-tax net MPK curve is lower than the before-tax curve. The steady-state capital stock occurs where the after-tax net MPK curve intersects $\rho$. This capital stock, $k_{1}$, is smaller than the steady-state capital stock without taxes, $k_{0}$. At $k_{1}$, the before-tax net MPK (which equals the before-tax interest rate) can be determined from the original MPK $-\delta$ curve. From this curve, we see that a lower capital stock implies a higher before-tax net MPK.

According to our simple neoclassical growth model, a higher tax rate on income from capital causes the economy to move to a steady state with a lower capital stock, lower output, and lower consumption. Notice that the growth rate remains $n+g$. Some more recent and more complex models suggest that a higher capital income tax rate may lower the long-run growth rate of the economy by reducing the rate of technological progress, $g$. To the extent that technological progress results from investment in human capital, a higher labor income tax rate may have similar effects. A higher tax rate on labor earnings reduces the after-tax return on investment in education, training, and other forms of capital.



Figure 1


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