

Notes on Exchange Rates, Purchasing Power Parity,  
and Interest Rate Parity

A. Nominal Exchange Rate

The nominal exchange rate is the rate of exchange between two currencies. We will adopt the convention of defining the nominal exchange rate as units of foreign currency per unit of home currency. Thus, if the United States is the home country and Germany is the foreign country, the nominal exchange rate is Deutsche Marks per dollar (DM/\$). The symbol for the nominal exchange rate is  $e$ . When defined in this way, an increase in  $e$  indicates an appreciation or strengthening of the home currency.

B. Law of One Price

The law of one price states that the price of a commodity, when stated in terms of any given currency, should be the same everywhere. Define

$P^a$  = price of a bushel of wheat in the United States (America), quoted in dollars

$P^b$  = price of a bushel of wheat in Britain, quoted in pounds

$e$  = nominal exchange rate in £/\$.

The price in British pounds of a bushel of American wheat is  $eP^a$ , and the law of one price states that

$$P^b = eP^a. \quad (1)$$

The law of one price is a theoretical proposition that may or may not describe the real world. The strongest intuitive argument in favor of the law of one price concerns arbitrage. If equation (1) does not hold, then there may be profit opportunities to buy a good in one country and sell it in another. Conditions that make arbitrage difficult may lead to violations of the law of one price. These conditions include tariffs, quotas or other trade restrictions, and transportation costs.

C. Absolute Purchasing Power Parity

Suppose the law of one price holds for every commodity. If we construct a basket of U.S. commodities and an identical basket of British commodities, then equation (1) continues to hold, but now the prices refer to the dollar price and the pound price of the entire commodity basket.

The theory of Absolute Purchasing Power Parity (Absolute PPP) says that equation (1) holds for baskets of commodities when the prices are stated in terms of the two different currencies. In our current example, this theory states that one U.S. dollar (or one British pound) has the same purchasing power over U.S. goods as over British goods.

Like any theory, absolute PPP may or may not be a good description of the real world. Violations of the law of one price, for any of the reasons cited above, lead to violations of absolute PPP. In addition, absolute PPP may not hold if the two commodity baskets are different, even if the law of one price holds for each commodity separately. Suppose, for example, that tea makes up a large share of the British commodity basket while coffee is more important in the American basket. If the price of coffee increases relative to the price of tea, the cost of the American basket increases relative to the cost of the British basket, leading to an apparent violation of absolute PPP. If such relative price changes are large and persistent enough, absolute PPP may not be a good description of actual economies.

#### D. Real Exchange Rate

The nominal exchange rate is the amount of foreign currency that can be obtained for one unit of domestic currency. The real exchange rate is the number of units of foreign goods that can be obtained for one unit of domestic goods.

How can we calculate the real exchange rate between the United States and Britain? Start with one unit of U.S. goods and carry out the following sequence of transactions:

1. Sell the U.S. goods for  $\$P^a$ .
2. Exchange these dollars at the nominal exchange rate  $e$  to obtain  $\pounds eP^a$ .
3. Use the pounds to buy  $eP^a/P^b$  units of U.K. goods at a price of  $P^b$  per unit.

This sequence of transactions implies that the real exchange rate, or the number of units of U.K. goods per unit of U.S. goods is

$$\epsilon = \frac{eP^a}{P^b}. \quad (2)$$

If absolute PPP holds, then the real exchange rate is equal to unity. I.e., equation (1) implies that  $\epsilon = 1$ . If  $\epsilon < 1$ , then U.S. goods are said to be undervalued because one unit of U.S. goods exchanges for less than one unit of U.K. goods. If  $\epsilon > 1$ , then U.S. goods are said to be overvalued.

Deviations of the real exchange rate from unity indicate deviations from absolute PPP.

If absolute PPP holds in the long run but not at every instant, then the real exchange rate does not always equal unity, but it tends to revert to unity over time. In other words, deviations from absolute PPP are not permanent but tend to disappear over time.

If absolute PPP does not hold even in the long run, the real exchange rate can wander arbitrarily far away from unity and does not tend to return to unity. In other words, deviations from absolute PPP are permanent and do not tend to disappear over time.

## E. Exchange Rates and Inflation Rates

Restating equation (2) in growth rates gives

$$\% \Delta \epsilon = \% \Delta e + \pi^a - \pi^b, \quad (3)$$

where  $\pi^a$  and  $\pi^b$  are the U.S. and U.K. inflation rates.

If PPP holds at all times, then the real exchange rate never changes ( $\% \Delta \epsilon = 0$ ) and

$$\% \Delta e = \pi^b - \pi^a. \quad (4)$$

Under a system of floating exchange rates, where  $e$  is free to vary and is not fixed by either of the governments, PPP predicts that the dollar appreciates over time if the U.S. inflation rate is lower than the U.K. inflation rate. Under a system of fixed exchange rates, PPP predicts that a one-time devaluation of the dollar ( $\% \Delta e < 0$ ) causes the inflation rate in the United States to exceed that in Britain.

If PPP holds only in the long run and the real exchange rate  $\epsilon$  is currently above its long-run level, then long-run PPP predicts that  $\epsilon$  will eventually decline. According to equation (3), a decline in  $\epsilon$  in turn implies that the nominal exchange rate  $e$  will decline, that the U.K. inflation rate will exceed the U.S. inflation rate, or both.

If PPP does not hold even in the long run, then we cannot use equation (3) to make any useful predictions.

## F. Relative Purchasing Power Parity

In practice, we have almost no readily available data on the absolute dollar price of a U.S. basket of goods or the absolute pound price of a British basket of goods. Instead we have index numbers such as the Consumer Price Index (CPI) or the Producer Price Index (PPI). A price index measures the price of a basket of goods in a given year not in absolute terms, but relative to the price of that basket in some base year. By construction, the price index is set equal to 100 in the base year. The value of the price index in any other year depends on the base year chosen. For example, the U.S. CPI in 1995 is 150 using 1983 as the base year. In other words, consumer prices rose 50 percent between 1983 and 1995. Using 1950 as the base year, however, the CPI in 1995 is 600.

Without observing absolute price levels, we cannot tell whether absolute PPP holds or not. This is because price indexes can take on very different levels, depending on which base years are used for the two countries. We can, however, use the weaker concept of Relative Purchasing Power Parity. Instead of equation (1), relative PPP states that

$$\alpha P^b = e P^a, \quad (5)$$

where the factor of proportionality  $\alpha$  depends on the base years chosen for the two price indices. For any choice of base years,  $\alpha$  is a fixed number.

If relative PPP holds continuously, then equation (5) implies that the real exchange rate is always equal to  $\alpha$ . If relative PPP holds only in the long run, then the real exchange rate does not always equal  $\alpha$ .

but it tends to revert to  $\alpha$  over time. In other words,  $\alpha$  is the long-run mean level of the real exchange rate.

Note that equation (3) continues to hold and can still be used to make predictions. For example, if relative PPP holds continuously, then the real exchange rate is always equal to  $\alpha$  and never changes, so that equation (4) still holds. If relative PPP holds only in the long run and the real exchange rate  $\epsilon$  is currently above its normal long-run level  $\alpha$ , then long-run PPP predicts that  $\epsilon$  will eventually decline. From equation (4), a decline in  $\epsilon$  in turn implies that the nominal exchange rate  $e$  will decline, that the U.K. inflation rate will exceed the U.S. inflation rate, or both.

Empirical evidence indicates that relative PPP does not hold continuously, but it seems to hold in the long run. Deviations from relative PPP are sufficiently long-lasting that equation (3) is not useful for predicting short-term movements in exchange rates. This equation does have predictive power concerning long-run movements in real exchange rates. By itself, however, the model cannot predict how these real exchange rate movements will be divided between nominal exchange rate movements and price level changes. For many purposes, only the real exchange rate matters and the division into nominal exchange rates and price levels is unimportant.

### G. Spot and Forward Exchange Rates

The nominal exchange rate  $e$  introduced above is known as the spot rate. It is the rate that applies to currency trades that take place at the same time the two parties agree to make the trade. It is also possible to make an agreement today to exchange currency at some fixed time in the future. The rate of exchange and the amount of currency to be traded are agreed upon today. The exchange rate that applies to such transactions is known as the forward rate, denoted  $f$ .

It is sometimes assumed that the forward rate observed today is equal to today's expectation of the spot rate that will occur in the future. Using our notation, this assumption states that

$$f_{t+k} = E_t(e_{t+k}), \quad (6)$$

where

$E_t(\cdot)$  = expected value based on information available today (i.e., at time  $t$ )

$e_{t+k}$  = spot rate that will occur  $k$  periods in the future

$f_{t+k}$  = today's forward rate for currency to be delivered  $k$  periods in the future.

Empirical evidence indicates that the forward rate is not generally a very accurate predictor of the future spot rate, and it may not always be an unbiased predictor.

### H. Interest Rate Parity

Interest Rate Parity (IRP) is a theory that describes the relation between nominal exchange rates and domestic and foreign nominal interest rates.

To understand IRP, compare the following two investment strategies:

Strategy A: Invest \$100 in American assets with interest rate  $i^a$ .

Dollar payoff at time  $t+1$ :  $\$100(1 + i^a)$ .

Strategy B: Invest \$100 in British assets with interest rate  $i^b$ .

This strategy entails the following sequence of transactions:

1. Buy  $\pounds 100e_t$  of British currency today.
2. Invest in British assets with payoff  $\pounds 100e_t(1 + i^b)$ .
3. Convert this payoff into dollars at the future spot rate  $e_{t+1}$ .

Dollar payoff at time  $t+1$ :  $\$100e_t(1 + i^b)/e_{t+1}$ .

Uncovered IRP states that the dollar payoffs on these two investments should be the same, which implies that

$$1 + i^b = (1 + i^a)e_{t+1}/e_t = (1 + i^a)(1 + \% \Delta e). \quad (7)$$

For typical values of interest rates and exchange rate changes, equation (7) is approximately equivalent to

$$\% \Delta e = i^b - i^a. \quad (8)$$

For example, if the British interest rate is 10% and the U.S. interest rate is 6%, equation (8) implies that the dollar appreciates 4% in value relative to the pound. Thus, an American who invests in British assets earns a 10% return in pounds but loses 4% on the round-trip currency transaction, leaving a rate of return of only 6% after converting back into dollars. This return is equivalent to that from investing in U.S. assets.

Assuming no default risk, the two nominal interest rates are known with certainty at the time the investment is made. Thus, the payoff on the investment in American assets is certain at time  $t$ . The payoff on the British investment is uncertain, however, because the future spot exchange rate is currently unknown. For this reason, the uncovered IRP condition is generally written in terms of the expected change in the exchange rate:

$$E(\% \Delta e) = i^b - i^a. \quad (9)$$

The investment position of an American buying British assets is said to be "uncovered" because this investment is subject to foreign exchange risk.

It is possible to eliminate this foreign exchange risk by using forward contracts. Rather than converting pounds back into dollars at whatever spot exchange rate happens to prevail at time  $t+1$ , an American investor can enter into a forward contract to sell pounds at time  $t+1$  at a forward rate agreed

upon today. The sequence of transactions for an American investor is the same as that outlined above, except that the forward exchange rate replaces the future spot rate in the calculations. With this change, equation (9) becomes

$$\frac{f_{t+1} - e_t}{e_t} = i^b - i^a. \quad (10)$$

The quantity  $(f_{t+1} - e_t)/e_t$  is called the forward premium. Equation (10) is called the covered IRP condition because the American investor's foreign exchange risk has been covered or eliminated in the forward market. The covered IRP condition states that the forward premium is equal to the interest rate differential between the two countries.