Empirical Performance of Lévy Option Pricing Models*

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Abstract

There are a number of recent models that extend the Black and Scholes (1973) model by considering stochastic volatility and/or jumps, and appear to show good empirical performance. In this paper we consider some of the most successful models, all of them belonging to the class of Lévy processes, and further study their empirical performance; in particular we consider their pricing performance for American options and their performance in terms of their put-call robustness; we find that their performance is good on the call side, but their put-call robustness gets lower scores than Black and Scholes (1973), with the possible exception of Carr, Geman, Madan, and Yor (2002); we interpret our results as evidence of overfitting.

Keywords: Lévy Process, Fourier Transform

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Abstract

There are a number of recent models that extend the Black and Scholes (1973) model by considering stochastic volatility and/or jumps, and appear to show good empirical performance. In this paper we consider some of the most successful models, all of them belonging to the class of Lévy processes, and further study their empirical performance; in particular we consider their pricing performance for American options and their performance in terms of their put-call robustness; we find that their performance is good on the call side, but their put-call robustness gets lower scores than Black and Scholes (1973), with the possible exception of Carr, Geman, Madan, and Yor (2002); we interpret our results as evidence of overfitting.

1 Introduction

The path breaking work of Black and Scholes (1973) started one of the most prolific areas of research in economics. Option pricing models have a direct application to one of the fastest growing segments of the financial markets, and that explains the amount of resources devoted to this problem. The Black and Scholes (1973) model (BS) remains the paradigm of option pricing and the benchmark against which all other models and extensions are compared. In addition, there are some obvious drawbacks in BS, and that has guided some of the extensions. Two of the main problems are the following: (i) BS ignores possible price jumps of the underlying; (ii) BS assumes constant volatility across moneyness and maturity. Both of these are clearly rejected by many empirical studies as, for example, Rubinstein (1985) and Bakshi, Cao, and Chen (1997).

To address the former of these problems, Merton (1976) proposed a jump diffusion model (MJD), under which the log return has both a diffusion component and a jump component. Furthermore, the jump component is assumed to be a compound Poisson jump process, so that a closed form solution for European option prices exists. However, Poisson jumps happen at a very "slow" rate, that is, imply finite activity. A further extension of this approach is the model introduced in Madan and Seneta (1990) and Madan, Carr, and Chang (1998); this is the so called variance gamma model (VG), under which the jump component of the log return is an infinite activity process. A generalization of all the previous is the model formalized in Carr, Geman, Madan, and Yor (2002) (CGMY) -although the idea first appeared Koponen (1995)- which allows diffusions and jumps of both finite and infinite activity. Daal and Madan (2005) and Huang and Wu (2004) further argued that in the infinite activity jump models, the diffusion component of log return is redundant, so all these models can be simplified to pure jump models. Among other efforts to model jump processes, we mention Barndorff-Nielsen (1998), Duffie, Pan, and Singleton (2000), Kou and Wang (2004), Carr and Wu (2004).

A simultaneous (and sometimes related) strand of the literature relaxes the assumption of constant volatility and has also developed quickly. The more direct way to introduce stochastic volatility into the option pricing models is to build stochastic models for the local diffusion volatility. Heston (1993) generalized BS by assuming a square-root, mean-reverting process for the diffusion volatility. The Heston (1993) model (H) has very good mathematical tractability, because the characteristic function of the local diffusion volatility process can be derived analytically. Bates (1996) builds on H to generalize MJD and comes up with a stochastic volatility model with jumps (SVJ). He uses SVJ to estimate options in the Deutsche Mark market. Finally, another approach to model stochastic volatility which provides enough flexibility to capture volatility clustering in the market, is to assume some stochastic process and substitute it for calendar time in a given dynamic model. Carr, Geman, Madan, and Yor (2003) consider the integral (to make sure it is increasing) of a mean-reverting square-root diffusion process ("economic time") and substitute it for the calendar time of a continuous-time underlying process; in particular, they apply it to the variance-gamma process of Madan and Seneta (1990) and Madan, Carr, and Chang (1998) (we will call this model VGSA) and to the generalization of Carr, Geman, Madan, and Yor (2002) (CGMYSA).

Complementary to the previous papers, there is also a large empirical literature that tests the performance of the "enhanced" models. Most of these papers use index options or currency options. Bates (1991) studies the behavior of MJD before the 1987 crash, and shows that the S&P 500 option market experienced crash fears before October, 1987. Lam, Chang, and Lee (2002) test the performance of VG in the Hang Seng index options market and conclude that VG marginally outperforms BS. Carr and Wu (2003a) and Carr and Wu (2003b) study whether a diffusion component is needed in the presence of high-activity log-stable processes, and find that there is no conclusive evidence that a diffusion term is necessary to model the behavior of the S&P 500 index.

In a parallel paper, Carr, Geman, Madan, and Yor (2002), using CGMY, also show that the index dynamics are devoid of a diffusion component. After the development of time changed Lévy models by Carr, Geman, Madan, and Yor (2003) and Carr and Wu (2004), Huang and Wu (2004) investigate their empirical performance in the S&P 500 index options market. They argue that a good model for the underlying index return dynamics should include a high frequency jump component, and should allow stochastic volatility to be independent of both the diffusion component and the jump component. Meanwhile, Eraker, Johannes, and Polson (2003) and Eraker (2004) argue that we can improve the empirical performance of SVJ by incorporating jumps in the volatility of return. On the other hand, Daal and Yu (2006) show that stochastic volatility does not improve the empirical performance of pure jump models in the S&P 500 index options market. Besides the index options market, a lot of empirical testing is conducted in the currency options market. As we mentioned before, Bates (1996) uses SVJ to estimate the parameters implicit in Deutsche mark options and tests for consistency with dollar/mark futures prices and the implicit volatility sample path. Daal and Madan (2005) compare the performance of the BS, MJD and VG in the Deutsche mark foreign currency options market. They find that VG has better out of sample performance and less entropy between the statistical measure and the risk neutral measure. More recently, Carr and Wu (2007) present evidence of stochastic skewness in over-the-counter currency option markets, and develop a class of models that capture the stochastic skewness. Their empirical tests show that their models outperform traditional models, both in-sample and out of sample. Bakshi, Carr, and Wu (2007) study stochastic risk premia and stochastic skewness in over-the-counter option markets with country dependent time changed Lévy models. Their analysis shows that risk premia are economically compatible with movements in stock and bond market fundamentals.

Overall, the extensions to BS show strong empirical performance, especially the time changed Lévy processes. Yet several fundamental questions still remain unanswered, before we can conclude that these are the models that we need to use in practice: (i) What is the performance of the new models for American options? Except for Bates (1996), the empirical tests use European options. However, in practice, most options are American style. (ii) Are the new models more put-call robust than BS, the benchmark? The ideal option pricing model should be robust to both call and put sides; that is, we should be able to calculate options prices in one side by applying parameters estimated from the other side. (iii) How many parameters are optimal to capture the dynamic process of underlying assets? BS only has one parameter (the constant volatility, σ), while the new models all have multiple parameters. With more parameters we can capture more dynamic properties, but we also bear the risk of data overfitting when we calibrate the model. So we need to find a tradeoff between these two effects.

In this paper we try to shed some light on these fundamental questions. For that purpose, we need to price both American calls and American puts and, in the process, estimate parameter values of the underlying. This is a non-trivial problem because of the lack of analytical results to price American options, which would allow the estimation of the parameter values in a relatively straightforward manner. In order to use numerical methods in a tractable way, we need to extend the Barone-Adesi and Whaley (1987) method which approximates the early exercise premium of American options under BS to the extensions we mentioned. We use S&P 100 index options, which are American options, and we perform the following exercises: (i) We analyze the in and out of sample performance of the different models; (ii) we use parameter values we estimate on the call side to price put options and compare with market prices, to study the call-put robustness of the models and the overfitting risk.

With respect to the call side, our tests confirm the results in the literature for European calls. We show that BS and H, which have the lowest number of parameters, are considerably outperformed by the other models with jumps. The performance of the other models with jumps depends on moneyness and maturity. However, our put side tests show that BS exhibits better call-put robustness performance than all other models, with the possible exception of CGMY. A possible explanation is that the other models overfit the dynamics of the underlying, and their apparent good performance on the call side is just the result of the time series autocorrelation on the prices.

The outline of this paper is as follows. In the second section, we explain the steps we follow in our analysis. In section 3 we introduce the models we consider in the paper and their basic properties. In section 4 we explain in detail the computational methodology. Section 5 presents the results of our tests. We close the paper with some conclusions.

2 Methodology

Our objective is to compare the pricing performance of some of the most widely used stochastic models for the underlying process of an option. We discuss these models in the next section. We are interested in American options, which is the standard type of options when the underlying is an individual stock, but also for some popular indices. However, analytical formulas do not exist in general and, from the option properties, puts are notoriously harder to price than calls. For that reason, the exercise we perform consists in first retrieving parameter values from call prices -in the way we later explain-, then we use these values to price the puts and, finally, we compare these put prices to the data. Once we estimate parameter values from call prices, it is possible to price puts numerically with high accuracy. However, in order to empirically estimate parameter values from call prices, we need a very efficient pricing method. This is a sketch of the general algorithm we use for our comparison of different models:

- First, we use the Lévy-Khintchine representation to derive the closed-form characteristic function for each of the models. We will derive the characteristic function in the next section, when we introduce the models.

- Then, we use the Fourier transform method, that we will describe later (see, for example Carr and Madan, 1999) to price the European call price corresponding to each model.

- In order to derive the price of the corresponding American option we need to calculate the early exercise premium. For that purpose, we use a modification -this is an innovation in this paper- of the quadratic approximation method of Barone-Adesi and Whaley (1987). This method, as well as the modification we introduce in this paper, is discussed later.

- Armed with a fast and yet relatively accurate -as we show later- method to price the American call option, we use Maximum Likelihood Estimation to derive the different model parameter values from actual prices.

- We use the parameters so derived to price the American put, using the method presented in Ibáñez and Zapatero (2004).

- We compare the prices so derived with the prices observed in the data.

In the coming sections we will follow the steps of this algorithm.

3 Underlying Models

In the introduction we reviewed some of the extensions to BS and their empirical performance. As we argued there, overall, these models show superior empirical performance than BS, but the test are restricted by the most part to European calls. We plan to test some of the most successful models for American options and for their put-call robustness. For the algorithm that we have just described there are two critical pieces of information that we need from each model to be tested: i) the characteristic function of the underlying, so that we can use the inverse Fourier transform to price the call option; ii) an algorithm that will allow us to simulate the underlying, so that we can price numerically the American put. We derive and/or discuss both of them for each of the models we plan to test.

3.1 Constant volatility models

3.1.1 Model of Black and Scholes (1973) (BS)

In BS the price of the underlying, $S(t)$, follows a geometric Brownian motion with constant drift r and volatility σ,

$$\frac{dS}{S} = (r - q)dt + \sigma dW(t) \tag{1}$$

where r is the riskless interest rate, q is the dividend yield, and $W(t)$ is a Brownian motion process.

Solving the previous stochastic differential equation we derive the following price for the underlying,

$$S(t) = S(0)exp\left((r - q - \frac{1}{2}\sigma^2)t + \sigma W(t)\right) \tag{2}$$

The underlying $S(t)$ satisfies a log-normal distribution. We introduce the following auxiliary stochastic variable,

$$X_t = log\left(\frac{S_t}{S_0}\right) - (r - q)t. \tag{3}$$

Its characteristic function is

$$\phi_X(u) = E[e^{iuX}] = exp\left(-\frac{1}{2}u(u+i)\sigma^2 t\right). \tag{4}$$

The simulation of the underlying is straightforward from the discrete-time version of equation (2,

$$S(t+\Delta t) = S(t)exp\left((r-q-\frac{1}{2}\sigma^2)\Delta t + \sigma W(\Delta t)\right), \tag{5}$$

where $W(\Delta t)$ is a draw from a normal distribution with mean zero and standard deviation $\sqrt{\Delta t}$.

3.1.2 Model of Merton (1976) (MJD)

In addition to the pure diffusion component, MJD allows a compound Poisson jump in the log return; in addition, MJD assumes that the log stock price jump size follows a normal distribution,

$$\frac{dS}{S} = (r-q-\lambda k)dt + \sigma dW(t) + (y(t)-1)dq(t) \tag{6}$$

where,

$$log(y(t)) \sim N(\mu, \delta^2)$$

$$k = E(y(t)-1) = e^{\mu+\frac{1}{2}\delta^2} - 1$$

$$P(dq(t)=1) = \lambda dt \qquad P(dq(t)=0) = 1-\lambda dt$$

for $dt \rightarrow 0$.

The price of the underlying can be expressed as,

$$S(t) = S(0)exp\left((r - \frac{\sigma^2}{2} - \lambda k)t + \sigma W(t) + \sum_{i=1}^{N_t} Y_i\right) \tag{7}$$

where N_t is a Poisson process with density λ.

The Lévy measure for the jump component is $\pi(dx) = \frac{\lambda}{\sqrt{2\pi\delta^2}} exp(-\frac{(x-\mu)^2}{2\delta^2})$. By applying the Lévy-Khintchine representation, the closed-form characteristic function for X_t, as defined in (3), is given by,

$$\begin{aligned}
\phi_X(u) &= exp\left\{iu\omega t - \frac{1}{2}u^2\sigma^2 t + t\int [e^{iux} - 1]\pi(dx)dx\right\} \\
&= exp\left\{iu\omega t - \frac{1}{2}u^2\sigma^2 t + t\int [e^{iux} - 1]\frac{\lambda}{\sqrt{2\pi\delta^2}} exp\{-\frac{(x-\mu)^2}{2\delta^2}\}dx\right\} \\
&= exp\left\{iu\omega t - \frac{1}{2}u^2\sigma^2 t + \lambda t\left(e^{iu\mu - u^2\delta^2/2} - 1\right)\right\} \qquad (8)
\end{aligned}$$

where ω has to be such that $\phi_X(-i) = 1$.

To simulate the stock process under MJD model during time interval [0, T], we can first evenly divide the time interval into N parts, so that each subinterval Δt, $\Delta t = \frac{T}{N}$, is very small. We assume that during Δt the jump probability is $\lambda \Delta t$. For a given stock price at time t_i, the simulation process is as follows.

1. Generate a random variable $u_{1i} \sim U[0, 1]$;

2. If $u_{1i} < \lambda \Delta t$, generate a random variable $u_{2i} \sim N(\mu, \delta^2)$, otherwise set $u_{2i} = 0$;

3. Generate random variable $u_{3i} \sim N(0, \Delta t)$;

4. $S(t_{i+1}) = S(t_i)exp\left((r - \frac{\sigma^2}{2} - \lambda k)\Delta t + \sigma u_{3i} + u_{2i}\right)$.

3.1.3 Model of Madan and Seneta (1990) and Madan, Carr and Chang (1998) (VG)

The variance gamma (or VG) process is defined as a time-changed Brownian motion process, in which the time increment follows a gamma process. Specifically,

$$Y(t) = \theta_{VG}\gamma(t) + \sigma W(\gamma(t)) \tag{9}$$

where, $\gamma(t) := \gamma(t; 1, \nu)$, is a gamma process with unit mean rate.

The price of the underlying under VG can be written as,

$$S(t) = S(0)exp\left((r-q)t + Y(t;\sigma,\nu,\theta_{VG}) + \omega t\right) \tag{10}$$

The characteristic function of X_t, defined in (3), is,

$$\phi_X(u) = \frac{e^{iu\omega t}}{\left(1 - i\theta_{VG}\nu u + \frac{1}{2}\sigma^2\nu u^2\right)^{\frac{t}{\nu}}} \tag{11}$$

Since $\phi_X(-i) = 1$, we have,

$$\omega = \frac{1}{\nu}log\left(1 - \theta_{VG}\nu - \frac{1}{2}\sigma^2\nu\right) \tag{12}$$

Alternatively, we can consider the VG process as a Lévy pure jump process with Lévy measure,

$$k_{VG}(x) = \begin{cases} \frac{Cexp(Gx)}{|x|} & x < 0 \\ \frac{Cexp(-Mx)}{x} & x > 0 \end{cases} \tag{13}$$

where,

$$C = \frac{1}{\nu} \tag{14}$$

$$G = \left(\sqrt{\frac{\theta_{VG}^2 \nu^2}{4} + \frac{\sigma^2 \nu}{2}} - \frac{\theta_{VG} \nu}{2} \right)^{-1} \tag{15}$$

$$M = \left(\sqrt{\frac{\theta_{VG}^2 \nu^2}{4} + \frac{\sigma^2 \nu}{2}} + \frac{\theta_{VG} \nu}{2} \right)^{-1} \tag{16}$$

Accordingly, the characteristic function for X_t can be written as,

$$\phi_X(u) = e^{iu\omega t} \left(\frac{GM}{GM + (M - G)iu + u^2} \right)^{Ct} \tag{17}$$

It can be shown that $Y(t; \sigma, \nu, \theta_{VG}) + Y(\Delta t; \sigma, \nu, \theta_{VG})$ and $Y(t + \Delta t; \sigma, \nu, \theta_{VG})$ follow the same distribution so, for a given S_i, we can simulate S_{i+1} by the following procedure:

1. Generate $\gamma(\Delta t) \sim \gamma(\Delta t; 1, \nu)$;

2. Generate $W(\gamma(\Delta t)) \sim N(0, \gamma(\Delta t))$;

3. Calculate $Y(\Delta t; \sigma, \nu, \theta_{VG}) = \theta_{VG} \gamma(\Delta t) + \sigma W(\gamma(\Delta t))$;

4. Calculate $S_{i+1} = S_i exp\left((r - q)\Delta t + Y(\Delta t; \sigma, \nu, \theta_{VG}) + \omega \Delta t\right)$.

3.1.4 Model of Carr, Geman, Madan and Yor (2002) (CGMY)

The Lévy measure of the CGMY process is,

$$k_{CGMY}(x) = \begin{cases} \frac{Cexp(Gx)}{|x|^{1+Y}} & x < 0 \\ \frac{Cexp(-Mx)}{x^{1+Y}} & x > 0 \end{cases} \tag{18}$$

The price of the underlying is,

$$S(t) = S(0) exp\left((r - q)t + \omega t + Z(t; C, G, M, Y)\right) \tag{19}$$

where $Z(t; C, G, M, Y)$ is a Lévy process, with Lévy measure given in equation (18).

The characteristic function for X_t, as defined in (3), is,

$$\phi_X(u) = exp(iu\omega t)exp\left(tC\Gamma(-Y)[(M-iu)^Y + (G+iu)^Y - M^Y - G^Y]\right) \tag{20}$$

where $\omega = -C\Gamma(-Y)[(M-1)^Y + (G+1)^Y - M^Y - G^Y]$.

There is no intuitive way to simulate the CGMY process. So far the most efficient and robust method is provided in Madan and Yor (2006). Their approach is to treat the CGMY process as a subordinated Brownian motion process and simulate the subordinator by the rejection method. Without giving a detailed derivation, we summarize their algorithm as follows: As in the previous models, the simulation method assumes that the price of the underlying S_i is known, and the time step is Δt.

1. Set $A = \frac{G-M}{2}, B = \frac{G+M}{2}, k = \frac{C\sqrt{\pi}}{\Gamma(\frac{Y+1}{2})2^{\frac{Y}{2}}}, \lambda = \frac{2k}{Y\epsilon^{\frac{Y}{2}}}, d = \frac{k\epsilon^{1-\frac{Y}{2}}}{1-\frac{Y}{2}}$, where, $\epsilon = 0.000001$;
2. Set $poi = 0, sum = d * \Delta t$;
3. Do until $poi > \Delta t$:
   (a) Generate $a \sim exponential(\lambda)$;
   (b) Set $poi = poi + a$;(if $poi > \Delta t$, break.)
   (c) Generate $u_1 \sim U[0,1], u_3 \sim U[0,1]$;
   (d) Set $y = \frac{\frac{\epsilon}{2}}{u_1^{\frac{2}{Y}}}$;
   (e) if $h(y,B,Y)exp\left(\frac{A^2y}{2}\right) > u_3, sum = sum + y$,
   where, $h(y,B,Y) = exp\left(\frac{b^2y}{2}\right)\frac{\Gamma\left(\frac{Y+1}{2}\right)}{\sqrt{\pi}}U(\frac{Y}{2},\frac{1}{2},\frac{B^2y}{2})$, and $U(a,b,x)$ is the confluent hypergeometric function;
4. Generate $u_2 \sim N(0,1)$;
5. Set $Z(\Delta t) = A * sum + u_2\sqrt{sum}$;
6. Set $S_{i+1} = S_i exp\left((r-q)\Delta t + \omega\Delta t + Z(\Delta t)\right)$;

3.2 Stochastic Volatility Models

3.2.1 Model of Heston (1993) (H)

This is the most widely used stochastic volatility model. It assumes that the instantaneous variance follows a square root mean-reverting process that we will call CIR process, since it was made popular in Cox, Ingersoll, and Ross (1985). Specifically,

$$\begin{aligned} &\tfrac{dS}{S} = (r-q)dt + \sqrt{V}_t(\rho dW^1(t) + \sqrt{1-\rho^2}dW^2(t) \\ &dV_t = \kappa(\theta - V_t)dt + \sigma_\lambda \sqrt{V}_t dW^1(t) \end{aligned} \tag{21}$$

where $W^1(t), W^2(t)$ are independent Brownian motion processes.

The characteristic function for X_t, as defined in (3), is,

$$\phi_X(u) = exp\left(C(u) + D(u)V_0\right), \tag{22}$$

where,

$$C(u) = \frac{\theta\kappa}{\sigma_\lambda^2}\left((\kappa - iu\rho\sigma_\lambda - \alpha)t - 2log\left(\frac{1-\beta e^{-\alpha t}}{1-\beta}\right)\right) \tag{23}$$

$$D(u) = \frac{(\kappa - iu\rho\sigma_\lambda - \alpha)\left(1 - e^{-\alpha t}\right)}{\sigma_\lambda^2\left(1 - \beta e^{-\alpha t}\right)} \tag{24}$$

and,

$$\alpha = \sqrt{(iu\rho\sigma_\lambda - \kappa)^2 - \sigma_\lambda^2(-iu - u^2)} \tag{25}$$

$$\beta = \frac{\kappa - iu\rho\sigma_\lambda - \alpha}{\kappa - iu\rho\sigma_\lambda + \alpha} \tag{26}$$

The price of the underlying is,

$$\begin{aligned} S(t) &= S(0)exp\left((r-q)t - \frac{1}{2}\int_0^t V_s ds + \rho\int_0^t \sqrt{V_s}dW^1(s)\right) \\ &\quad *exp\left(\sqrt{1-\rho^2}\int_0^t \sqrt{V_s}dW^2(s)\right) \end{aligned} \tag{27}$$

For simulation purposes, we follow a two-step scheme, (i) simulate V_t, and (ii) simulate S_t.

The CIR process for stochastic volatility can be exactly simulated by the method described in Broadie and Kaya (2006). As Cox, Ingersoll, and Ross (1985) observe, the transition law of V_t given $V_u(u < t)$ is,

$$V_t = \frac{\sigma_\lambda^2 \left(1 - e^{-\kappa(t-u)}\right)}{4\kappa} \chi_d^{\prime 2} \left(\frac{4\kappa e^{-\kappa(t-u)}}{\sigma_\lambda^2 \left(1 - e^{-\kappa(t-u)}\right)} V_u \right) \tag{28}$$

where $\chi_d^{\prime 2}(\lambda)$ is the non-central chi-squared distribution with d degrees of freedom, and non-centrality parameter λ, and

$$d = \frac{4\theta\kappa}{\sigma_\lambda^2}. \tag{29}$$

The non-central chi-square distribution can be represented as an ordinary chi-square distribution with random degrees of freedom. Specifically, χ_{d+2N}^2 and $\chi_d^{\prime 2}(\lambda)$ follow the same distribution, if N follows a Poisson distribution with mean $\frac{1}{2}\lambda$.

For simulation purposes, we discretize according to Milstein method because this method has a high order of convergence and it is easy to implement. Based on equation (21), the discretization implied by Milstein algorithm is,

$$\begin{aligned} S_{i+1} &= S_i + S_i(r-q)\Delta + S_i\sqrt{V_i}(\rho\Delta W^1(t) + \sqrt{1-\rho^2}\Delta W^2(t)) \\ &\quad + \frac{1}{2} S_i V_i \{(\rho\Delta W^1(t) + \sqrt{1-\rho^2}\Delta W^2(t))^2 - \Delta t\} \end{aligned} \tag{30}$$

Therefore, for any price of the underlying S_i, we generate two i.i.d. random variables, $u_1, u_2 \sim N(0, \Delta t)$, and substitute them into the last equation to calculate the stock price for next moment, S_{i+1}.

3.2.2 Model of Bates (1996) (SVJ)

This model can be regarded as a combination of MJD and H. More explicitly, this model is characterized by the following equations,

$$\begin{aligned}&\tfrac{dS}{S} = (r-q-\lambda k)dt + \sqrt{V_t}(\rho dW^1(t) + \sqrt{1-\rho^2}dW^2(t) + (e^{\mu+\delta Z}-1)dN(t)\\ &dV_t = \kappa(\theta - V_t)dt + \sigma_\lambda\sqrt{V_t}dW^1(t)\end{aligned} \tag{31}$$

The price of the underlying is

$$\begin{aligned} S(t) &= S(0)exp\left((r-q-\lambda k)t - \frac{1}{2}\int_0^t V_s ds + \rho\int_0^t \sqrt{V_s}dW^1(s)\right)\\ &\quad *exp\left(\sqrt{1-\rho^2}\int_0^t \sqrt{V_s}dW^2(s) + \sum_{i=1}^{N_t} Y_i\right) \end{aligned} \tag{32}$$

Since the jump part and the diffusion part are independent, the characteristic function for X_t, defined in (3), is,

$$\phi_X(u) = exp\left\{C(u) + D(u)V(0) + \lambda t[(e^{(iu\kappa - \frac{1}{2}u^2\delta^2)} - 1) - iu(e^{\kappa+\frac{\delta^2}{2}} - 1)]\right\} \tag{33}$$

where $C(u)$ and $D(u)$ are as in equations (23) and (24).

The simulation of SVJ requires a slight modification of the simulation method for H. For a given price at time t_i, to simulate the price of the underlying at t_{i+1}, we do as follows,

1. Generate $u_{1i} \sim U[0,1]$, and two i.i.d. random variables $u_{2i}, u_{3i} \sim N(0, \Delta t)$;
2. If $u_{1i} < \lambda\Delta t$, generate $u_{4i} \sim N(\mu, \delta^2)$, otherwise set $u_{4i} = 0$;
3. Set $\Delta W^1(\Delta t) = u_{2i}, \Delta W^2(\Delta t) = u_{3i}$, and calculate an intermediate S_{i+1} using equation (30)
4. $S_{i+1} = S_{i+1}exp\,(u_{4i} - \lambda k\Delta t)$.

3.2.3 First model of Carr, Geman, Madan and Yor (2003) (VGSA)

Both VG and CGMY are pure jump models. There is no diffusion component in the log return, so we cannot introduce stochastic volatility as in H and SVJ. Carr, Geman, Madan, and Yor (2003) and Carr and Wu (2004) develop a time-changed method to introduce stochastic volatility. VGSA assumes that the increasing rate of the "economic time" follows a mean-reverting square root (or CIR) process,

$$dy = \kappa(\theta - y)dt + \sigma_\lambda \sqrt{y} dW \tag{34}$$

And the economic clock is given by,

$$Y(t) = \int_0^t y(u)du \tag{35}$$

The characteristic function for Y(t) is,

$$\begin{aligned} \phi_Y(u) &= \phi_Y(u, t, y(0); \kappa, \theta, \sigma_\lambda) \\ &= A(t, u) exp\left(B(t, u)y(0)\right) \end{aligned} \tag{36}$$

where,

$$\begin{aligned} &A(t,u) = \frac{exp\left(\frac{\kappa^2\theta t}{\sigma_\lambda^2}\right)}{\left(cosh(\frac{\gamma t}{2}) + \frac{\kappa}{\gamma} sinh(\frac{\gamma t}{2})\right)^{2\kappa\theta/\sigma_\lambda^2}} \\ &B(t,u) = \frac{2iu}{\kappa + \gamma coth\left(\frac{\gamma t}{2}\right)} \\ &\gamma = \sqrt{\kappa^2 - 2\sigma_\lambda^2 iu} \end{aligned} \tag{37}$$

Substituting calendar time by economic time in the VG process, we can get the time-changed VG process,

$$Z_{VG}(t) = X_{VG}(Y(t); C, G, M) \tag{38}$$

where, $X(t; C, G, M)$ represents a standard VG process.

The characteristic function of the time-changed VG process can be derived in the following way,

$$\begin{aligned}\phi_{VGSA}(u) &= E\left[exp(iuZ_{VG}(t)\right] \\ &= E\left[\psi_{VG}(u; 1, G, M)Y(t)\right] \\ &= \phi_Y(-i\psi_{VG}(u; 1, G, M), t, C; \kappa, \theta, \sigma_\lambda) \qquad (39)\end{aligned}$$

For this derivation we re-scale parameters following Carr, Geman, Madan, and Yor (2003). In addition, $\psi_{VG}(u; C, G, M)$ is the unit time log characteristic function of the standard VG process. Specifically,

$$\phi_{VG}(u) = exp\left(t\psi_{VG}(u)\right) \qquad (40)$$

The price of the underlying at time t is,

$$S(t) = S(0)\frac{exp\left((r-q)t + Z_{VG}(t)\right)}{E[exp(Z_{VG}(t))]} \qquad (41)$$

So the characteristic function of X_t, as defined in (3), is,

$$\phi_X(u) = \frac{\phi_{VGSA}(-i\psi_{VG}(u; 1, G, M), t, C; \kappa, \theta, \sigma_\lambda)}{\phi_{VGSA}(-i\psi_{VG}(-i; 1, G, M), t, C; \kappa, \theta, \sigma_\lambda)^{iu}} \qquad (42)$$

The simulation of the price of the underlying requires a two-step strategy. First we need to simulate the "economic clock." We do this by simulating a stochastic integral ($Y(t) = \int_0^t y(t)dt$). Then we need to simulate the price process by the same method we used to simulate the standard VG process. Here is the step-by-step algorithm to simulate a price process during the time interval $[0, T]$, with initial value $S(0)$:

1. Divide T evenly into N intervals, so that we simulate stock prices on the grid points, $S_1, S_2, ..., S_N$.

2. Simulate the economic time on the grid points.

   (a) For better accuracy, we divide each interval from step 1 into M parts, so that each subinterval has length $\frac{T}{MN}$;

   (b) Apply the exact simulation method we discussed in subsection 2.2.1 to simulate $y(t)$ on all MN grid points;

   (c) Numerically, integrate $y(t)$ over each interval to get the economic time on the grid points, $S_1, S_2, ..., S_N$.

3. Starting at time 0, use the method introduced in 2.1.3 to simulate the VG process $Z_{VG}(Y(t_1)), Z_{VG}(Y(t_2)), ..., Z_{VG}(Y(t_N))$. Instead of Δt, we now use ΔY for the time interval.

4. Substitute Z_{VG} from step 3 into equation (41) to calculate the prices.

3.2.4 Second model of Carr, Geman, Madan and Yor (2003) (CGMYSA)

The time clustering mechanism for CGMYSA model is also a CIR process, defined in (21). Similar to the VGSA process, the CGMYSA process can be constructed by replacing calendar time with economic time in the standard CGMY process.

$$Z_{CGMY} = X_{CGMY}(Y(t); C, G, M, Y) \tag{43}$$

The characteristic function for the CGMYSA process is,

$$\phi_{CGMYSA}(u) = \phi_Y(-i\psi_{CGMY}(u; 1, G, M, Y), t, C; \kappa, \theta, \sigma_\lambda) \tag{44}$$

where, again, $\psi_{CGMY}(u; C, G, M, Y)$ is the unit time log characteristic function of the standard CGMY process.

The price of the underlying in CGMYSA is

$$S(t) = S(0)\frac{exp\left((r-q)t + Z_{CGMY}(t)\right)}{E[exp(Z_{CGMY}(t))]} \tag{45}$$

The characteristic function of X_t, as defined in (3), is,

$$\phi_{CGMYSA}(u) = \frac{\phi(-i\psi_{CGMY}(u; 1, G, M, Y), t, C; \kappa, \theta, \sigma_\lambda)}{\phi(-i\psi_{CGMY}(-i; 1, G, M, Y), t, C; \kappa, \theta, \sigma_\lambda)^{iu}} \tag{46}$$

The simulation of this model is similar to that of VGSA. The only change is in step 2(b) of the algorithm, in which we need to simulate the standard CGMY process, instead of the VG process.

4 Data and Econometric Methodology

4.1 Data

We use data on prices of the S&P 100 option. This is one of the most active option contracts, and the options are American style. In addition, the dividend yield for the S&P 100 is available. We use data from Jan 1, 2002 to Dec 31, 2003. The data includes transaction date, expiration date, closing price, strike price, volume, zero coupon rate during the option period, call ask price, call bid price, put ask price and put bid price. The annualized monthly dividend rate is retrieved from WRDS. We use the midpoint of ask and bid price as the spot price of the option. We exclude observations according to the following criterium: if the bid-ask spread of the option is more than thirty percent of the average of ask and bid prices, we eliminate the observation because the midpoint might be a noisy estimate of the true value of the option. Also, since almost all the observations we exclude correspond to short term out-of-the-money options, we reduce liquidity-related biases. We end with 20440 observations on the call side and 26285 observations on the put side.

Next we separate the option data into different categories according to moneyness and maturity. As shown in the following tables, we divide call option moneyness into five bins and put option moneyness into six bins according to the ratio of spot price and strike price. For call options, the four grid points are 0.95, 0.985, 1.015 and 1.05, while for put options the five grid points are 0.95, 0.985, 1.015, 1.05 and 1.085, respectively. For call options, 6940 observations are deep out of the money($S/K \in (0, 0.95)$), 4236 observations are out of money($S/K \in [0.95, 0.985)$), 3599 observations are at the money($S/K \in [0.985, 1.015)$), 2881 observations are in the money($S/K \in [1.015, 1.05)$) and 2784 observations are deep in the money ($S/K \in [1.05, \infty)$). For put options, 3309 observations are deep in the money, 3316 observations are in the money, 3832 observations are at the money, 4022 observations are out of the money, 3330 observations are deep out of the money and 8476 observations are far out of the money ($S/K \in (1.085, \infty)$). We have more bins for the put options than for call option because there are more put observations than call observations, and a large proportion of the put options are out of money ($S/K > 1.015$). With respect to maturity, we classify an option as a short-term option if it has less than 30 days to expiration, as a medium-term option if the time to expiration is 30 to 90 days, and as a long-term option when it has more than 90 days to expiration. Based on this classification, we have 6989 short-term, 10261 medium-term, and 3190 long-term call options. For put options, we have 8310 short-term, 13316 medium-term, and 4659 long-term options.

Table 1: Call options sample properties

Moneyness (S/K)	Days-to-Expiration <= 30	30–90	> 90	Subtotal
< 0.95	1.72 (0.20) 1186	3.65 (0.44) 3898	7.20 (0.80) 1856	6940
0.95–0.985	3.25 (0.26) 1503	7.54 (0.64) 2271	21.38 (2.00) 462	4236
0.985–1.015	6.27 (0.41) 1432	14.64 (0.99) 1801	23.69 (1.80) 366	3599
1.015–1.05	15.94 (1.06) 1383	23.75 (1.60) 1237	33.31 (2.02) 261	2881
>= 1.05	36.30 (1.96) 1485	42.96 (2.06) 1054	52.71 (1.97) 245	2784
Subtotal	6989	10261	3190	20440

4.2 Econometric Methodology

We follow two steps: first we estimate parameters for each model from call prices, and test the in-sample and out-of-sample performance of each model; second, we use the parameter values estimated from the call side to price the put options. We achieve two objectives: From the call side, where the estimation is easier (as we discuss next), we study the over-fitting effect of the different multi-parameter models. Also, we test the put-call consistency of each model.

We estimate the call side parameters for each model via maximum likelihood. The likelihood function is as in Madan, Carr, and Chang (1998). This involves calculating the price of the American options for each parameter set, Θ. To compute the price of the American options we split their value in two components, the value of the corre-

Table 2: Put options sample properties

Moneyness(S/K)	Days-to-Expiration			Subtotal
	$<= 30$	30–90	> 90	
< 0.95	39.53 (2.04) 1389	69.76 (2.36) 1257	83.12 (2.13) 663	3309
0.95–0.985	17.03 (1.10) 1481	25.10 (1.65) 1427	36.15 (2.02) 408	3316
0.985–1.015	7.12 (0.45) 1437	14.68 (0.94) 1912	26.64 (1.95) 483	3832
1.015–1.05	3.82 (0.30) 1462	9.72 (0.74) 2057	19.24 (1.46) 503	4022
1.05–1.085	2.63 (0.27) 1099	6.26 (0.59) 1777	15.22 (1.32) 454	3330
$>= 1.085$	1.64 (0.21) 1442	3.37 (0.42) 4886	6.06 (0.68) 2148	8476
Subtotal	8310	13316	4659	26285

For each box, three numbers are reported. They are the average of ask-bid mid-point prices, average of ask-bid spread (in parentheses) and number of observations, respectively.

sponding European option, and the early exercise premium. Since most of the models we study do not have analytic probability distribution functions, but have closed form characteristic functions, we calculate prices of the European options via the Fourier transform method. In order to calculate the early exercise premium of the American options under the infinite activity Lévy models, we extend the quadratic approximation method of Barone-Adesi and Whaley (1987), and test numerically its performance. For the put side test, we use Monte Carlo simulation to calculate the prices of the American put options.

Details of the different methodologies are provided in the following sections.

4.2.1 Option Pricing via the Generalized Fourier Transform

The models we discussed in section 2 do not have closed form probability distributions for the price of the underlying at a future time T, except BS. However, they all have closed form characteristic functions. A large number of papers develop applications of the Fourier transform to address this particular set of cases. Here, we focus on the approach followed by Carr and Madan (1999) and Lewis (2000).

The starting point is the standard pricing equation for an European call option under the risk-neutral probability measure,

$$C_T(k) = e^{-rT}E[(S_T - K)^+] = \int_k^{\infty} e^{-rT}(e^s - e^k)q_T(s)ds, \tag{47}$$

where $s = log(S_T)$ and $k = log(K)$.

If it is possible to compute the Fourier transform (or characteristic function) of the last equation, then we can derive the price of the call option through the inverse transformation. In a first step, we multiply both sides of the equation by a factor

$exp(\alpha k)$ to guarantee that the Fourier transform converges. That is,

$$c_T(k) = exp(\alpha k) C_T(k) = exp(\alpha k) \int_k^{\infty} e^{-rT}(e^s - e^k) q_T(s) ds \tag{48}$$

Carr and Madan (1999) show that if $E[S_T^{\alpha+1}] < \infty$, the characteristic function for $log(S_T)$, $\phi_{log(S_T)}(v - (\alpha+1)i)$, is finite for any real number v. It is then possible to derive the Fourier transform of $c_T(k)$ as follows,

$$\begin{aligned}
\psi_c(v) &= \int_{-\infty}^{\infty} e^{ivk} \int_k^{\infty} e^{\alpha k} e^{-rT}(e^s - e^k) q_T(s) ds dk \\
&= \int_{-\infty}^{\infty} e^{-rT} q_T(s) \int_{-\infty}^{s} (e^{s+\alpha k} - e^{(1+\alpha)k}) e^{ivk} dk ds \\
&= \int_{-\infty}^{\infty} e^{-rT} q_T(s) \left(\frac{e^{(\alpha+1+iv)s}}{\alpha + iv} - \frac{e^{(\alpha+1+iv)s}}{\alpha + 1 + iv} \right) ds \\
&= \frac{e^{-rT} \phi_{log(S_T)}(v - (\alpha+1)i)}{\alpha^2 + \alpha - v^2 + i(2\alpha+1)v}
\end{aligned} \tag{49}$$

Here we can see why we need a dump factor α. If we set $\alpha = 0$, $\psi_c(v)$ is infinite at $v = 0$, and we cannot compute $C_T(k)$ through the inverse Fourier transform.

Using the inverse Fourier transform of equation (48), we derive the European call option price,

$$C_T(k) = \frac{exp(-\alpha k)}{2\pi} \int_{-\infty}^{\infty} e^{-ivk} \psi_c(v) dv = \frac{exp(-\alpha k)}{\pi} \int_0^{\infty} e^{-ivk} \psi_c(v) dv \tag{50}$$

A refinement of the previous method to calculate option prices through the Fourier transform is presented in Lewis (2000), and studied in Itkin (2005). An advantage of this method is that it relies on analytical results as much as possible, which makes it computationally more efficient. In this paper we use the idea of Lewis (2000) to calculate option prices.

This method also starts with the standard pricing equation for an European call option under the risk-neutral probability measure,

$$C_T(x_0, K) = e^{-rT} \int_{-\infty}^{\infty} (e^x - K)^+ q(x, x_0, T) dx \tag{51}$$

Lewis (2000) observes that the last integral can be calculated using Parseval's identity,

$$\int_{-\infty}^{\infty} f(x) g(x) dx = \frac{1}{2\pi} \int_{i\nu-\infty}^{i\nu+\infty} \hat{f}(z) \hat{g}(z) dz \tag{52}$$

where the hat functions are Fourier transforms, which we need to calculate next. We need also to point out that the identity holds only on the region where both $\hat{f}(z)$ and $\hat{g}(z)$ are regular.

With respect to the first part of (51), the payoff function of a call option, we can find its Fourier transform by simple integration,

$$\hat{\omega}(z) = \int_{\infty}^{\infty} e^{izx} (e^x - K)^+ dx = -\frac{K e^{iz \log(K)}}{z^2 - iz}, \qquad Imz > 1 \tag{53}$$

We then can calculate the Fourier transform of $q(x, x_0, T)$, the second factor in the integral of (51), from the characteristic function of X_T. We recall our definitions of variables, $X_0 = \log(S_0), X_T = \log(S_T) - \log(S_0) - (r - q)T$. The Fourier transform of the probability density function, $q(x, x_0, T)$, is related to the characteristic function in the following way,

$$\begin{aligned} \hat{q}(z, x_0, T) &= \phi_{\log(S_T)}(z) = E\left[e^{iu \log(S_T)}\right] \\ &= e^{iz(\log(S_0)+(r-q)T)} E\left[e^{izX_T}\right] = e^{iz(\log(S_0)+(r-q)T)} \phi_X(z) \end{aligned} \tag{54}$$

With the Fourier transforms of the payoff function and the density function, we can apply Parseval's identity to calculate the price of the call option,

$$\begin{aligned} C_T(S_0, K) &= -\frac{Ke^{-rT}}{2\pi}\int_{i\nu_1-\infty}^{i\nu_1+\infty} e^{-iz(\log(S_0)+(r-q)T)}\phi_X(-z)e^{iz\log(K)}\frac{dz}{z^2-iz} \\ &= -\frac{Ke^{-rT}}{2\pi}\int_{i\nu_1-\infty}^{i\nu_1+\infty} e^{-izk}\phi_X(-z)\frac{dz}{z^2-iz} \end{aligned} \tag{55}$$

where $k = \log(\frac{S_0}{K}) + (r-q)T$, and ν_1 is in the strip where both $\hat{\omega}(z)$ and $\phi_X(-z)$ converge. For convenience, we denote by $\mathcal{S}_\omega$ and $\bar{\mathcal{S}}_X$ the regions where $\hat{\omega}(z)$ and $\phi(-z)$ are regular, respectively. By this notation, $\mathcal{S}_\omega = \{z : Imz \in (1,\infty)\}$. On the other hand, $\bar{\mathcal{S}}_X$ is model dependent, and it is not unique for each model. However, $\phi(-z)$ is regular on $\{z : Imz \in (a,b)\}$, with $a < 0$ and $b > 1$, for all the models we are discussing here, so we can always get the strip where the Parseval's identity holds. That is, the set $\mathcal{S}_\omega \bigcap \bar{\mathcal{S}}_X = \{z : Imz \in (1,b)\}$, is not empty. We can always find a ν_1 such that $0 < \nu_1 < b$.

In our $\bar{\mathcal{S}}_X$, the only two singularities for the integrand in equation (55) are $z = 0$ and $z = i$. The pole for $z = i$ has a residue $Se^{-qT}i/(2\pi)$. To reduce the computational load, we can move the integration contour to $\nu_2 \in (0,1)$. By the residue theorem, the call option value must also equal the integral along $Imz = \nu_2$ minus $2\pi i$ times the residue at $z = i$. For symmetry, we choose $\nu_2 = \frac{1}{2}$ and get,

$$\begin{aligned} C_T(S,K) &= Se^{-qT} - \frac{Ke^{-rT}}{2\pi}\int_{\frac{1}{2}i-\infty}^{\frac{1}{2}i+\infty} e^{-izk}\phi_T(-z)\frac{dz}{z^2-iz} \\ &= Se^{-qT} - \frac{1}{\pi}\sqrt{SK}e^{-(r+q)T/2}\int_0^\infty Re\left[e^{iuk}\phi_T\left(u-\frac{i}{2}\right)\right]\frac{du}{u^2+\frac{1}{4}} \end{aligned} \tag{56}$$

It is straightforward to compute the previous integral numerically.

4.2.2 A Modified Quadratic Method to Approximate Values of American Options Under Lévy Models

Finite difference methods are too costly to price American options. In addition, they are impractical with stochastic volatility. In the previous subsection we discussed the methodology to price European options using the Fourier transform. In this subsection we study the computation of the early exercise premium that, added to the price of the European option, will yield the price of the American option. For that purpose, we modify the quadratic approximation method of Barone-Adesi and Whaley (1987) to calculate the early exercise premium for the jump models as well as the stochastic volatility models. We first review this method, which they introduce for the case in which the underlying follows BS.

We denote by $\epsilon(S,T)$ the early exercise premium,

$$\epsilon(S,T) = C(S,T) - c(S,T) \tag{57}$$

where $C(S,T)$ is the price of the American option and $c(S,T)$ is the price of the European option. The early exercise premium satisfies the following partial differential equation,

$$\frac{1}{2}\sigma^2 S^2 \epsilon_{SS} - r\epsilon + (r-q)S\epsilon_S - \epsilon_T = 0 \tag{58}$$

We set $M = 2r/\sigma^2$ and $N = 2(r-q)/\sigma^2$, and we rewrite the last equation,

$$S^2\epsilon_{SS} - M\epsilon + NS\epsilon_S - \frac{M}{r}\epsilon_T = 0 \tag{59}$$

From equation (59), we observe that if ϵ were only a function of T, it would be

$$\epsilon(S,T) = L(T) = 1 - e^{-rT} \tag{60}$$

We then set $\epsilon_C(S, L(T)) = L(T)f(S, L(T))$. It therefore follows that $\epsilon_{SS} = L(T)f_{SS}$ and $\epsilon_T = L_T f + LL_T f_K$. Equation (59) becomes,

$$S^2 f_{SS} + NSf_S - \frac{M}{L}f - (1-L)Mf_L = 0 \tag{61}$$

Now we need to make an approximation. We make the last term in equation (61) equal to 0. The argument is that as T approaches 0, f_L also approaches 0, while as L goes to infinity, $L(T)$ goes to 1. Equation (61) becomes an ordinary differential equation,

$$S^2 f_{SS} + NSf_S - \frac{M}{L}f = 0 \tag{62}$$

This equation can be solved analytically, and the solution is,

$$f(S) = a_1 S^{q_1} + a_2 S^{q_2} \tag{63}$$

where

$$q_1 = \frac{-(N-1) - \sqrt{(N-1)^2 + 4M/L}}{2} \tag{64}$$

$$q_2 = \frac{-(N-1) + \sqrt{(N-1)^2 + 4M/L}}{2} \tag{65}$$

From the boundary conditions for ϵ, we know that $a_1 = 0$. So we can express the price of the American call option as,

$$C(S,T) = c(S,T) + La_2 S^{q_2} \tag{66}$$

Now the only unknown is a_2. We can solve for a_2 by considering the American option price on the optimal exercise boundary. By definition of optimal boundary, the following two equations must be satisfied,

$$S^* - X = c(S^*, T) + La_2 S^{*q_2} \tag{67}$$

and the smooth pasting condition,

$$1 = e^{-qT} N[d_1(S^*)] + Lq_2 a_2 S^{*q_2 - 1} \tag{68}$$

where, $d_1(S^*) = \frac{log \frac{S^*}{X} + \left((r-q) + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$. Thus, we have two equations and two unknowns. From equation (68), we can isolate a_2,

$$a_2 = \frac{1 - e^{-qT} N[d_1(S^*)]}{K q_2 S^{*q_2 - 1}} \tag{69}$$

We substitute equation (69) in (67) to get an expression for S^*,

$$S^* - X = c(S^*, T) + \{1 - e^{-qT} N[d_1(S^*)]\} S^*/q_2 \tag{70}$$

We can solve this equation numerically. With S^* known, we can calculate a_2 from (69). After some simplifications, we can express the price of an American call option as

$$C(S,T) = \begin{cases} c(S,T) + A_2 (\frac{S}{S^*})^{q_2}, & if \quad S < S^* \\ S - X & if \quad S \geq S^* \end{cases} \tag{71}$$

where $A_2 = (S^*/q_2)\{1 - e^{-qT} N[d_1(S^*)]\}$.

Bates (1991) and Bates (1996) have extended this quadratic approximation method for MJD and SVJ. However, this extension does not work for Lévy models with infinite activity. Here we propose an approximation method and test it numerically under CGMY. Since VG is just a special case of CGMY, we do not test VG separately. The results of our test show that the error of our method is negligible, compared to the real value of the American options. By real value, we mean the value calculated through Monte Carlo simulation.

We assume that higher moments of the log return have negligible effect on the early exercise premium. So, ϵ still satisfies equation (59). We use quadratic variation as a proxy for the variance in this equation. Under CGMY, the quadratic variation is,

$$\tilde{\sigma}^2 = C\Gamma(2-Y)\left(\frac{1}{M^{2-Y}} + \frac{1}{G^{2-Y}}\right) \tag{72}$$

We can follow the same procedure as Barone-Adesi and Whaley (1987) to get a solution for the American option. Equation (67) still holds under CGMY, but we need to use equation (56) to calculate the value of the European option.

Equation (68) should be modified, because the first derivative of the price of the European option under CMGY is no longer $N[d_1(S)]$. We can calculate the new derivative by differentiating equation (56),

$$\frac{\partial c}{\partial S} = e^{-qT}\left(1 - \frac{1}{\pi}\sqrt{\frac{K}{S}}e^{-(r-q)\frac{T}{2}}\int_0^\infty Re\left[i(u-\frac{i}{2})e^{iku}\phi_{CGMY}(u-\frac{i}{2})\right]\frac{du}{u^2+\frac{1}{2}}\right) \tag{73}$$

Our solution for the American options is just a new version of equation (71), with A_2 replaced by $\tilde{A}_2 = (S^*/q_2)\left(1 - \frac{\partial c}{\partial S}\right)$.

Similar to Bates (1991) and Bates (1996), we use the expected quadratic variation as variance of the time changed Lévy models. Under CGMYSA, we can calculate the annualized variance as,

$$\begin{aligned}
\tilde{\tilde{\sigma}}^2 &= \frac{1}{T}\int_{-\infty}^{\infty} x^2\nu(dx)E\left[\int_0^T y(u)du\right] \\
&= \frac{\int_{-\infty}^{\infty} x^2\nu(dx)}{iT}\left[\frac{\partial}{\partial u}A(T,u)|_{u=0} + \frac{\partial}{\partial u}B(T,u)|_{u=0} \times y(0)\right]
\end{aligned} \tag{74}$$

Table 3: Comparison of American Option Values Under CGMY Model

Model Parameters	r	q	T(days)	S	K	Sim Result	Apr Result
C=7.5	0.03	0.07	30	500	460	41.99	42.00
G=16.5	0.03	0.07	30	500	480	25.68	25.71
M=52	0.03	0.07	30	500	500	12.73	12.72
Y=0.3	0.03	0.07	30	500	520	4.45	4.46
	0.03	0.07	30	500	540	1.05	1.04
C=0.13	0.05	0.05	90	500	460	50.68	50.72
G=12.0	0.05	0.05	90	500	480	37.59	37.79
M=17.0	0.05	0.05	90	500	500	27.26	27.21
Y=1.4	0.05	0.05	90	500	520	18.84	18.90
	0.05	0.05	90	500	540	12.74	12.70
C=0.05	0.07	0.03	120	500	460	56.11	56.15
G=0.5	0.07	0.03	120	500	480	40.41	40.47
M=30.0	0.07	0.03	120	500	500	26.59	26.66
Y=1.25	0.07	0.03	120	500	520	15.56	15.56
	0.07	0.03	120	500	540	7.84	7.79

$$= \frac{C}{\kappa T}\Gamma(2-Y)\left(\frac{1}{M^{2-Y}}+\frac{1}{G^{2-Y}}\right)\left[\theta(\kappa T-1+exp(-\kappa T))+\frac{2y(0)}{1+coth(\frac{\kappa T}{2})}\right]$$

The expression for $c(S,T)$ and $\frac{\partial c}{\partial S}$ should also be modified for CGMYSA. The modification is almost the same as for CGMY, except for the characteristic function $\phi_{CGMYSA}(u)$ instead of $\phi_{CGMY}(u)$.

Tables 3 and 4 report the results of our numerical test. The results show that our proposed approximation produces a mispricing error less than one percent of the simulated value. This is robust to a large range of parameter values, differents cost of carry, different (short) maturities, and different levels of moneyness. Considering the fact that the ask-bid spread counts at least 5 percents of the option value, our method appears very accurate. We will use this method to calibrate the Lévy models in the next subsection.

Table 4: Comparison of American Option Values Under CGMYSA Model

Model Parameters	r	q	T(days)	S	K	Sim Result	Apr Result
C=0.07	0.07	0.03	30	500	460	46.52	46.53
G=0.19	0.07	0.03	30	500	480	29.08	29.10
M=26.5	0.07	0.03	30	500	500	14.20	14.23
Y=1.3	0.07	0.03	30	500	520	4.63	4.63
$\kappa = 1.3$	0.07	0.03	30	500	540	1.00	0.99
$\theta = 0.004$							
$\sigma = 0.21$							
C=0.22	0.03	0.07	90	500	460	42.46	42.59
G=15.0	0.03	0.07	90	500	480	27.55	27.74
M=58.0	0.03	0.07	90	500	500	16.02	16.17
Y=0.7	0.03	0.07	90	500	520	8.11	8.21
$\kappa = 352$	0.03	0.07	90	500	540	3.48	3.54
$\theta = 1.25$							
$\sigma_\lambda = 6.30$							
C=2.1	0.05	0.05	120	500	460	53.51	53.59
G=9.3	0.05	0.05	120	500	480	40.22	40.37
M=51.0	0.05	0.05	120	500	500	29.13	29.14
Y=0.75	0.05	0.05	120	500	520	20.04	20.03
$\kappa = 18.0$	0.05	0.05	120	500	540	13.00	13.03
$\theta = 1.0$							
$\sigma_\lambda = 0.15$							

4.2.3 Risk Neutral Parameter Estimation

We can use a number of different objectives to calibrate option pricing models. Among them, the minimization of the absolute pricing error gives more weight to in the money, long term options, while the minimization of the relative error gives more weight to out of the money, short term options. Detlefsen and Hardle (2006) discuss in detail this problem and find that the results can vary substantially depending on the criterium. Here we settle this problem by using the Maximum Likelihood Estimation(MLE) method of Jacquier and Jarrow (1996), Elliott, Lahaie, and Madan (1997), and Madan, Carr, and Chang (1998). We let ω_i be the observed market price on the i-th option and $\hat{\omega}_i$ be the price according to the model under use; additionally, we assume that ω_i and $\hat{\omega}_i$ satisfy the following equation,

$$\omega_i = \hat{\omega}_i exp(\theta \varepsilon_i - \theta^2/2) \tag{75}$$

where $\varepsilon_i \sim N(0,1)$. The corresponding log likelihood function for ω_i is,

$$log(f(\Theta;\omega_i) = -\frac{1}{2}\sum_{i=1}^{M}\left(\frac{log(\omega_i) - log(\hat{\omega}_i)}{\theta} + \frac{\theta}{2}\right)^2 \tag{76}$$

where Θ is the parameter vector. Madan, Carr, and Chang (1998) show that maximum likelihood estimation is equivalent to the minimization of

$$k = \sqrt{\frac{1}{M}\sum_{i=1}^{M}(log(\omega_i) - log(\hat{\omega}_i))^2} \tag{77}$$

This is a non-linear minimization problem. We apply the standard Levenberg-Marquardt method to achieve the minimum point in the parameter space. To avoid local minima, we start with more than a hundred initial parameters guesses for each model.

4.2.4 Monte Carlo Simulation for American Option Pricing

In this paper, we want to investigate the put side performance of Lévy option pricing models. Since there is no closed form solution for the price of American style put options, we have to calculate it numerically. For constant volatility models, such as MJD, VG, and CGMY, we can derive a Partial Integro Differential Equation(PIDE) for the option price and solve it numerically by finite difference. However, for stochastic volatility models such as VGSA and CGMYSA, we cannot follow this approach to get option prices because we cannot derive a PIDE. An alternative approach we use in this paper is a modified version of Monte Carlo simulation. This approach is flexible, and its applicability does not depend on the dimension of the problem.

The basic idea is presented in Ibanez and Zapatero (2004), and is similar to the approximation we described above. It is based on the existence of an optimal exercise frontier, on which the "live value" of the American option is the same as the "intrinsic value.". The intrinsic value is the payoff to the holder if the option is exercised immediately, while the live value is the price of the American option resulting from following the optimal exercise policy (at the optimal frontier). On the optimal frontier,

$$P_t(V_t^*, K) = I_t(V_t^*, K) \tag{78}$$

where V_t is a M-dimensional state variable. We can set $V_t = (\bar{B}_t, S_t)$, where $\bar{B}_t \in R^{M-1}$ and $S_t \in R$. S_t stands for the stock price, and $\bar{B}_t$ may stand for all the other stochastic state variables which can affect the option price. If we just consider the one dimensional problem, $V_t = S_t$.

Given the initial time $t_0 = 0$, and maturity of the option T, we divide the time horizon evenly in N intervals, with exercise dates $t_1, t_2, ..., t_N$. By definition, the intrinsic value of the American option at t_n can be expressed as,

$$P_{tn}(S_{tn}, K) = e^{-r\tau} E^Q_{tn}\left[I_\tau(S_\tau, K)\right] \tag{79}$$

where τ is the optimal stopping time, defined as the first time $S_{t(n+i)} \le S^*_{t(n+i)}$, and $\tau = \infty$ otherwise.

The objective is to compute recursively the optimal exercise frontier surface at every exercise point. The algorithm is as follows:

1. Compute the optimal frontier surface at $t = t_{N-1}$;

   (a) Fix the values of the $M - 1$ parameters and compute the optimal exercise price for the M-th parameter. For each given $M - 1$ parameters set, $\bar{B}^i_{t(N-1)}$, we start from an initial value $S^1_{t(N-1)}$, and compute $S^2_{t(N-1)}$ as the solution to

   $$I_{t(N-1)}((\bar{B}^i_{t(N-1)}, S^2_{t(N-1)}), K) = P_{t(N-1)}((\bar{B}^i_{t(N-1)}, S^1_{t(N-1)}), K) \tag{80}$$

   We then compute $P_{t(N-1)}((\bar{B}^i_{t(N-1)}, S^2_{t(N-1)}), K)$, find S^3_t, and so on. We repeat this procedure s times, until we get a value $S^s_{t(N-1)}$ such that $|S^s_{t(N-1)} - S^{s-1}_{t(N-1)}| < \varepsilon$. We consider $S^s_{t(N-1)}$ as the solution to equation (78) and represent it as $S^{i*}_{t(N-1)}$.

   (b) Repeat step (a) for several more $\bar{B}^i_{t(N-1)}$, and use some interpolation and extrapolation techniques to expand the frontier from the grid points to the whole space. Thus, we get the an optimal exercise frontier surface for $t = t_{N-1}$.

2. Repeat step 1 recursively for $t_{N-2}, t_{N-3}, ..., t_0$ to get the optimal exercise frontier surface at the remaining exercise points.

3. After we have retrieved the optimal frontier surface, it is straightforward to compute the price of the American option using plain vanilla Monte Carlo simulation.

5 Empirical Analysis

There is a growing literature on testing the empirical performance of Lévy option pricing models. For instance, Lam, Chang, and Lee (2002), Daal and Madan (2005), Huang and Wu (2004) and Daal and Yu (2006). However, all those papers investigate the performance on the call side. The general conclusion is that Lévy option pricing models exhibit better calibration performance and better out of sample performance. Nevertheless, tests based only on the call side are not sufficient to draw the conclusion that more general Lévy models can explain market prices better than the traditional pure diffusion models. In this paper, we want to investigate the call side performance as well as the put side performance of each Lévy pricing model. As we will see, good performance on the call side does not necessarily translates into good performance on the put side, which raises doubts about the ability of the models to explain properly market prices.

5.1 Risk Neutral Model Calibration

Tables 5 and 6 report the average value of weekly parameter values estimates for each model. Parameter values are estimated as discussed in section 3. The diffusion volatility σ for BS is 0.22, while it is 0.12 for the MJD. This is not surprising since the volatility of MJD comes from both the diffusion and jump components. Similarly, we can see that the diffusion variance of H is higher than its counterpart in SVJ. In the finite jump models, MJD and SVJ, λ is the average number of jumps per year, and μ is the average jump size. Our calibration shows that in the constant volatility setting of MJD the average number of jumps is 35.70, while for the stochastic volatility setting of SVJ the average number of jumps is just 5.66. In contrast, the average jump size of MJD is -0.05, while the average jump size of SVJ is -0.17. This suggests that the small jumps in MJD are explained by the stochastic volatility in SVJ. With respect to the infinite activity jump models, recall that VG can be interpreted as a time changed BS and we can see that the diffusion volatility is almost the same as in BS, and the volatility for the Gamma process, ν, is very close to zero. Comparing CGMY and CGMYSA, we observe that parameters C, G, M, Y do not differ substantially: For CGMY, C, G, M, Y are equal to $11.14, 20.74, 48.40, 0.95$ respectively, while for CGMYSA they are equal to $11.14, 11.16, 37.19, 1.07$. A common property to all models is that they all suggest negative skewness, which is consistent with the conclusions of Madan, Carr, and Chang (1998) and Daal and Madan (2005). At the bottom of each table we report k, which is a measure of goodness of fit, defined in equation (77). Comparing all models, as expected, we find that the more parameters a model has, the smaller k is. The main question is: Do models with more parameters really explain better the behavior of the underlying, or does the improvement of the in-sample fit arise because more structural parameters overfit the market data? We answer this question in the following two subsections.

5.2 Call Side

As we have discussed, the empirical literature has focused on call prices. In this subsection we complement some of the results about call pricing performance. We report out of sample test results of the option pricing models applied to the call side. For each pricing model, we use the parameter values estimated with data from the previous week

Table 5: Risk Neutral Parameters for Constant Volatility models

Parameters	BS	MJD	VG	CGMY
σ	0.2242 (0.0637)	0.1208 (0.0930)	0.2286 (0.0578)	
λ		35.70 (32.32)		
μ		-0.0491 (0.1367)		
δ		0.0432 (0.0419)		
θ_{VG}			-0.8235 (0.6558)	
ν			0.0233 (0.0193)	
C				11.14 (28.57)
G				20.74 (27.08)
M				48.40 (28.60)
Y				0.94623 (0.5583)
k	0.0444 (0.0396)	0.0273 (0.0262)	0.0280 (0.0267)	0.0272 (0.0260)

Table 6: Risk Neutral Parameters for Stochastic Volatility models

Parameters	Heston	SVJ	VGSA	CGMYSA
θ	0.0807 (0.0415)	0.0648 (0.3535)	78.13 (130.14)	13.37 (31.85)
κ	105.71 (156.25)	108.85 (111.45)	100.80 (201.54)	102.8737 (194.20)
σ_λ	2.9446 (1.8444)	1.7850 (1.3740)	67.49 (109.66)	14.14 (54.91)
ρ	-0.3263 (0.4519)	-0.1198 (0.6486)		
V_0	0.0597 (0.0580)	0.0578 (0.0822)		
λ		5.6579 (9.3520)		
μ		-0.1673 (0.2475)		
δ		0.1372 (0.1714)		
C			82.45 (113.84)	11.14 (67.40)
G			43.19 (34.77)	11.16 (23.40)
M			76.50 (42.42)	37.19 (27.95)
Y				1.0660 (0.5415)
k	0.0251 (0.0249)	0.0224 (0.0242)	0.0203 (0.0202)	0.0203 (0.0209)

Table 7: Out-of-sample Pricing Errors for Each Model

Models	Absolute Errors	Absolute Percentage Errors
BS	0.9986(0.4926)	0.2245(0.1286)
MJD	0.7926(0.4707)	0.2033(0.1242)
VG	0.8207(0.4752)	0.2068(0.1189)
CGMY	0.7781(0.4215)	0.2007(0.1112)
Heston	0.8405(0.4703)	0.2312(0.1332)
SVJ	0.7699(0.4343)	0.2129(0.1221)
VGSA	0.7913(0.4624)	0.2055(0.1188)
CGMYSA	0.7746(0.4706)	0.2082(0.1242)

to calculate option prices for the current week. We compare model prices with observed market prices to calculate out-of-sample pricing errors. We report both the absolute pricing error and the absolute percentage pricing error. The absolute pricing error is the equal to the absolute difference between the observed market price and the model price, while the absolute percentage error is the absolute pricing error divided by the market price.

In Table 7 we report absolute and relative errors for all models. These are the average absolute pricing error and the average absolute percentage error for each week. The averages in this step are volume-weighted. We also report the standard deviations. We observe that BS has the largest absolute pricing error and second largest absolute percentage error, while H has the largest absolute percentage error and second largest absolute error. This unambiguously suggests that jumps are necessary in option pricing models, although it remains to be determined what type of jumps is best. From the same table we can see that the eight-parameter SVJ has the smallest absolute error and CGMY the smallest absolute percentage error. However, the differences are very small: SVJ only leads by less than 0.5 cents and CGMY by less than 0.2%. We cannot establish a clearly superior model based only on the out of sample weekly errors.

For a more in-depth examination of the pricing biases of each model, we compare them for different moneyness/maturity categories (moneyness is characterized by the ratio S/X. Tables 8- 15 report the results for the different models. We observe that the performance varies drastically with respect to moneyness. The absolute percentage errors for deep out of the money options are around 35%, and they decrease as the S/X increases. Absolute percentage errors for deep in the money options are around 5%. On the other hand, all the models have similar maturity biases in the in the money categories: Absolute percentage errors increase with the time to maturity when the option is in the money; but when for out of the money options, different models have different types of maturity biases.

In particular, we find that BS has the largest errors for almost all the mid-term and long term options, and H has largest errors for almost all the short term options. This explains why BS and H have the largest out of sample weekly errors. The bad performance of BS suggests that we need additional parameters to capture the skewness and kurtosis of the log return of the underlying. Among MJD, VG and CGMY, we observe that CGMY has the smallest absolute error and the smallest absolute percentage error, except for the deep in the money options, where MJD has the best performance. On the other hand, we find that stochastic volatility models perform better for long term options, but worse for short term and medium term options, compared to constant volatility models. This result is also consistent with many other papers.

Table 8: Errors According to Maturity and Moneyness for BS

Models	Moneyness	Call Options Days to Expiration		
	(S/K)	($\leq$ 30 days)	(30 $\sim$ 90 days)	($>$ 90 days)
BS	$X < 0.95$	0.5499 (0.6023)	1.0282 (1.2048)	2.5406 (2.3467)
		0.3711 (0.3372)	0.3199 (0.3712)	0.5132 (0.6618)
	$0.95 \sim 0.985$	0.6682 (0.7509)	1.3967 (1.2422)	4.5729 (2.3792)
		0.2537 (0.3517)	0.1784 (0.1188)	0.2148 (0.1012)
	$0.985 \sim 1.015$	0.8374 (0.8218)	2.2614 (1.5099)	5.7521 (2.1098)
		0.1973 (0.2993)	0.1530 (0.0832)	0.2526 (0.0946)
	$1.015 \sim 1.05$	1.3496 (1.0895)	3.1939 (1.6772)	7.4399 (2.4797)
		0.0807 (0.0549)	0.1338 (0.0618)	0.2273 (0.0803)
	$X \geq 1.05$	1.8851 (1.2499)	4.7775 (2.2190)	8.6128 (2.5346)
		0.0538 (0.0372)	0.1119 (0.0451)	0.1527 (0.0457)

Table 9: Errors According to Maturity and Moneyness for MJD

Models	Moneyness	Call Options Days to Expiration		
	(S/K)	($\leq$ 30 days)	(30 $\sim$ 90 days)	($>$ 90 days)
MJD	$X < 0.95$	0.5693 (0.6250)	1.0025 (1.2061)	2.2143 (2.0933)
		0.3761 (0.3298)	0.3245 (0.3620)	0.4828 (0.6439)
	$0.95 \sim 0.985$	0.6681 (0.7396)	1.0408 (1.0532)	3.0605 (1.9116)
		0.2460 (0.3051)	0.1501 (0.1233)	0.1349 (0.0717)
	$0.985 \sim 1.015$	0.7396 (0.7527)	1.1980 (1.2619)	2.7050 (1.6990)
		0.1587 (0.2232)	0.0775 (0.0631)	0.1147 (0.0629)
	$1.015 \sim 1.05$	0.9557 (0.9564)	1.6749 (1.3962)	3.1828 (2.2518)
		0.0581 (0.0507)	0.0668 (0.0488)	0.0932 (0.0612)
	$X \geq 1.05$	1.2346 (1.1096)	2.3521 (1.5895)	6.0102 (2.5922)
		0.0361 (0.0340)	0.0555 (0.0381)	0.1056 (0.0454)

Table 10: Errors According to Maturity and Moneyness for VG

Models	Moneyness	Call Options Days to Expiration		
	(S/K)	($\leq$ 30 days)	(30 $\sim$ 90 days)	($>$ 90 days)
VG	$X < 0.95$	0.5554 (0.5907)	0.9722 (1.0536)	2.1396 (2.0747)
		0.3712 (0.3157)	0.3250 (0.3663)	0.4889 (0.6395)
	$0.95 \sim 0.985$	0.6787 (0.7200)	1.1073 (1.0460)	3.2356 (2.1058)
		0.2488 (0.2766)	0.1586 (0.1302)	0.1463 (0.0864)
	$0.985 \sim 1.015$	0.7480 (0.7270)	1.3376 (1.1629)	3.4589 (1.5682)
		0.1554 (0.1792)	0.0873 (0.0597)	0.1501 (0.0654)
	$1.015 \sim 1.05$	0.9395 (0.9018)	1.8715 (1.3407)	4.9547 (1.9119)
		0.0578 (0.0502)	0.0754 (0.0475)	0.1503 (0.0580)
	$X \geq 1.05$	1.3111 (0.9929)	2.7335 (1.4696)	6.5467 (2.1267)
		0.0382 (0.0304)	0.0657 (0.0347)	0.1164 (0.0387)

Table 11: Errors According to Maturity and Moneyness for CGMY

Models	Moneyness	Call Options Days to Expiration		
	(S/K)	$(\leq 30$ days)	$(30 \sim 90$ days)	$(> 90$ days)
CGMY	$X < 0.95$	0.5525 (0.5754)	0.8867 (0.9031)	1.8592 (1.5963)
		0.3701 (0.3068)	0.3011 (0.3203)	0.4186 (0.5105)
	$0.95 \sim 0.985$	0.6579 (0.6903)	0.9835 (0.9261)	2.9835 (2.1963)
		0.2446 (0.2858)	0.1448 (0.1198)	0.1296 (0.0796)
	$0.985 \sim 1.015$	0.7283 (0.7222)	1.1791 (1.1398)	2.6975 (1.6264)
		0.1567 (0.2064)	0.0770 (0.0585)	0.1125 (0.0568)
	$1.015 \sim 1.05$	0.9726 (0.9623)	1.5972 (1.4161)	3.3743 (2.2433)
		0.0592 (0.0516)	0.0636 (0.0497)	0.0994 (0.0592)
	$X \geq 1.05$	1.3053 (1.1787)	2.6547 (1.7796)	6.7480 (3.1179)
		0.0382 (0.0358)	0.0625 (0.0421)	0.1198 (0.0567)

Table 12: Errors According to Maturity and Moneyness for H

Models	Moneyness	Call Options Days to Expiration		
	(S/K)	$(\leq 30$ days)	$(30 \sim 90$ days)	$(> 90$ days)
Heston	$X < 0.95$	0.6041 (0.6738)	0.9230 (0.9372)	1.5647 (1.5961)
		0.4089 (0.3900)	0.2997 (0.3133)	0.2849 (0.3907)
	$0.95 \sim 0.985$	0.6717 (0.7289)	1.1665 (1.0638)	3.7445 (2.2023)
		0.2767 (0.3418)	0.1710 (0.1437)	0.1671 (0.0869)
	$0.985 \sim 1.015$	0.7960 (0.7543)	1.3019 (1.2041)	3.2175 (2.5183)
		0.2041 (0.3131)	0.0894 (0.0726)	0.1365 (0.0979)
	$1.015 \sim 1.05$	0.9496 (0.9824)	1.5502 (1.2648)	3.3062 (2.3316)
		0.0575 (0.0516)	0.0632 (0.0469)	0.0980 (0.0665)
	$X \geq 1.05$	1.2174 (1.0928)	2.3324 (1.2965)	5.4703 (3.2701)
		0.0354 (0.0336)	0.0559 (0.0307)	0.0984 (0.0589)

Table 13: Errors According to Maturity and Moneyness for SVJ

Models	Moneyness	Call Options Days to Expiration		
	(S/K)	($\leq$ 30 days)	(30 $\sim$ 90 days)	($>$ 90 days)
SVJ	$X < 0.95$	0.5577 (0.6043)	0.8471 (0.8083)	1.6351 (1.9361)
		0.3744 (0.3634)	0.2801 (0.2587)	0.2830 (0.3242)
	0.95 $\sim$ 0.985	0.6602 (0.6934)	0.9975 (0.9517)	2.9990 (1.8542)
		0.2626 (0.3104)	0.1517 (0.1348)	0.1376 (0.0848)
	0.985 $\sim$ 1.015	0.7537 (0.7203)	1.1618 (1.0700)	1.8497 (2.7753)
		0.1804 (0.2653)	0.0819 (0.0672)	0.0761 (0.1065)
	1.015 $\sim$ 1.05	0.8740 (0.8787)	1.1538 (1.0856)	1.8645 (1.9042)
		0.0541 (0.0479)	0.0490 (0.0424)	0.0549 (0.0548)
	$X \geq 1.05$	0.8695 (0.9591)	1.4174 (1.1485)	4.7326 (3.2161)
		0.0254 (0.0302)	0.0330 (0.0265)	0.0841 (0.0557)

Table 14: Errors According to Maturity and Moneyness for VGSA

Models	Moneyness	Call Options Days to Expiration		
	(S/K)	($\leq$ 30 days)	(30 $\sim$ 90 days)	($>$ 90 days)
VGSA	$X < 0.95$	0.5499 (0.6053)	0.8441 (0.8364)	1.2735 (1.0649)
		0.3674 (0.3474)	0.2737 (0.2453)	0.2127 (0.1706)
	0.95 $\sim$ 0.985	0.6715 (0.7350)	1.0582 (1.0018)	2.9992 (2.0601)
		0.2613 (0.2960)	0.1548 (0.1310)	0.1318 (0.0783)
	0.985 $\sim$ 1.015	0.7773 (0.7542)	1.1409 (1.0199)	1.9290 (1.4202)
		0.1736 (0.2059)	0.0787 (0.0599)	0.0817 (0.0524)
	1.015 $\sim$ 1.05	0.9706 (0.8633)	1.4451 (1.2110)	3.7714 (2.6008)
		0.0611 (0.0497)	0.0588 (0.0441)	0.1119 (0.0747)
	$X \geq 1.05$	1.0926 (0.9124)	2.3871 (1.4445)	6.7185 (2.6501)
		0.0319 (0.0292)	0.0568 (0.0329)	0.1184 (0.0472)

Table 15: Errors According to Maturity and Moneyness for CGMYSA

Models	Moneyness	Call Options Days to Expiration		
	(S/K)	($\leq$ 30 days)	(30 $\sim$ 90 days)	($>$ 90 days)
CGMYSA	$X < 0.95$	0.5461 (0.6003)	0.8459 (0.8244)	1.1994 (1.0140)
		0.3690 (0.3446)	0.2725 (0.2387)	0.2028 (0.1713)
	0.95 $\sim$ 0.985	0.6660 (0.7295)	1.0415 (0.9890)	2.8119 (1.9878)
		0.2600 (0.3060)	0.1517 (0.1267)	0.1235 (0.0739)
	0.985 $\sim$ 1.015	0.7859 (0.7566)	1.1896 (1.0421)	1.7564 (1.1825)
		0.1848 (0.2450)	0.0827 (0.0630)	0.0743 (0.0466)
	1.015 $\sim$ 1.05	0.9354 (0.8887)	1.2412 (1.1860)	2.7868 (1.9328)
		0.0595 (0.0512)	0.0520 (0.0471)	0.0823 (0.0528)
	$X \geq 1.05$	0.8927 (0.9287)	1.7284 (1.3691)	5.0997 (3.7164)
		0.0260 (0.0302)	0.0400 (0.0309)	0.0906 (0.0657)

5.3 Put Side

As we argued before, empirical studies of option prices with new models for the underlying have focused on calls. However, it is generally accepted that option price time series are highly serially correlated, and stochastic volatility has mean reverting properties. For these reasons, it is difficult to evaluate based only on empirical studies of the call side, up to what extent the apparent success of some of these models might be the result of overfitting, since most of these models require more parameters. In this subsection, we extend the empirical test to the put side to further analyze which model provides a better understanding of the behavior of the underlying. As we have explained, we use the parameters we have estimated from the call side to price the puts, according to the Monte Carlo simulation methodology we have reviewed. This subsection represents the main contribution of the paper.

As for the call side, weekly errors are reported in table 16. The absolute error and absolute percentage error for BS are 0.9794 and 28.01%, respectively. Both of them are the second smallest in their respective categories. The only model that has smaller mean absolute error and mean absolute percentage error is CGMY, with 0. 9698 and 25.40, respectively, but with considerably larger standard deviation than BS. All the other models have significantly larger errors. Furthermore, constant volatility models perform better than stochastic volatility models. Overall, BS seems to do a good job at capturing the underlying behavior. CGMY seems promising, but more tests are necessary. All the other models see to exhibit a strong call-put bias, that we define as the fact that call and put prices imply different model parameters, and might be related to overfitting.

Table 16: Average Weekly Put Side Errors for Each Model

Models	Absolute Errors	Absolute Percentage Errors
BS	0.9794(0.3561)	0.2801(0.0699)
MJD	1.3098(0.6907)	0.3878(0.2670)
VG	1.2155(0.6044)	0.3152(0.1647)
CGMY	0.9698(0.5296)	0.2540(0.1547)
Heston	1.3299(0.7189)	0.3188(0.1666)
SVJ	1.6980(0.8735)	0.5446(0.3675)
VGSA	1.5607(0.7309)	0.3815(0.2046)
CGMYSA	2.0713(0.8916)	0.6665(0.3113)

Figures 1 - 8 report histograms of the weekly average of absolute errors for each model. We observe that for BS they are all below \$2.0, with sixty-two weeks lower than \$1.0, and ten greater than \$1.5. For CGMY, all the average weekly errors are less than \$3.0, with seventy weeks lower than \$1.0, but seven greater \$2.0, which explains why it has a lower average error, but higher standard deviation. These two models have the most weeks with absolute error less than \$1.0. On the other hand, VG, MJD, Heston, SVJ, VGSA and CGMYSA have only 47, 40, 40, 29, 27 and 13 weeks with error less than \$1.0, respectively. Additionally, all models, except BS, have several weeks with absolute errors larger than \$2.0; for example, VG has eleven, CGMY seven, MJD seventeen, H nineteen, and CGMYSA is the worst case, with more than fifty. This clearly shows that models with more than one parameter tend to overfit market data on the call side, and perform worse than BS on the put side. The results for average weekly absolute percentage errors are similar, but we do not report them here.

Tables 17 - 24 report put side errors for the different models, depending on maturity and moneyness. As on the call side, all models demonstrate strong moneyness biases. For instance, let us consider CGMY, which has the best overall performance, according to average weekly errors: The average absolute and absolute percentage errors for the

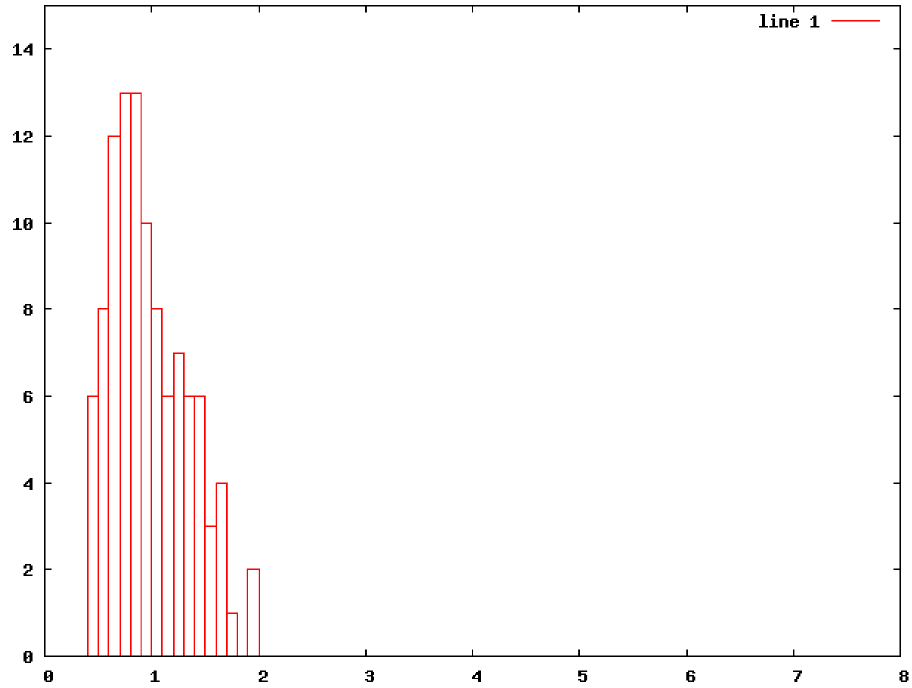

Figure 1: Absolute weekly errors for put testing under BS

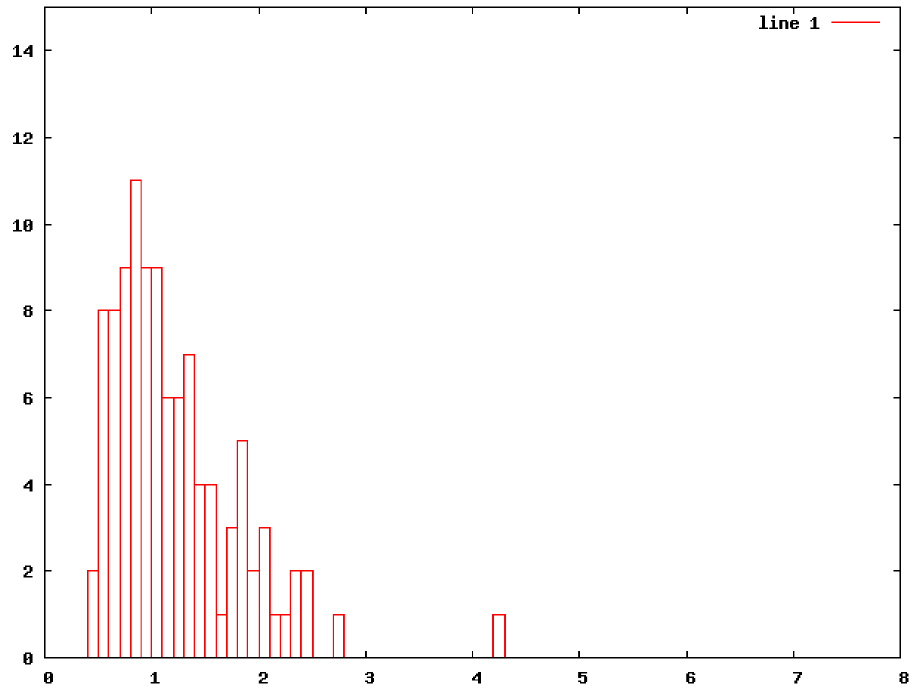

Figure 2: Absolue weekly errors for put testing under VG

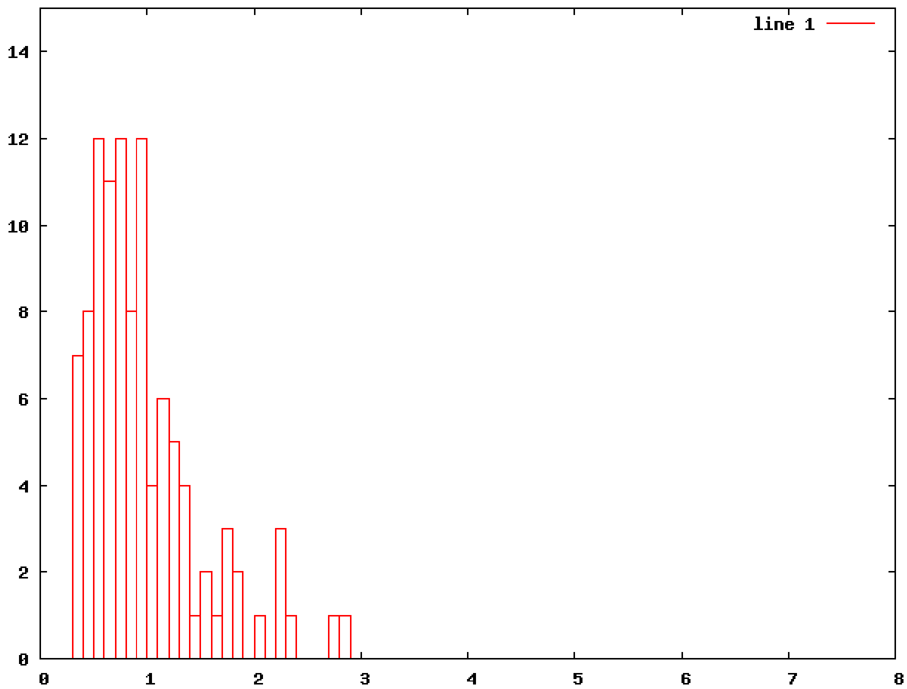

Figure 3: Absolute weekly errors for put testing under CGMY

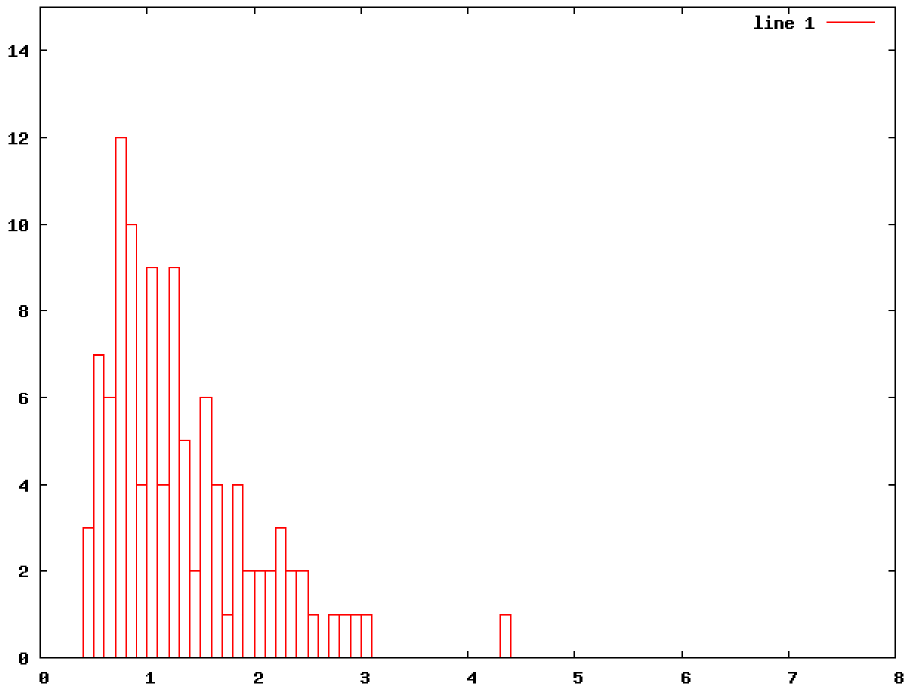

Figure 4: Absolute weekly errors for put testing under MJD

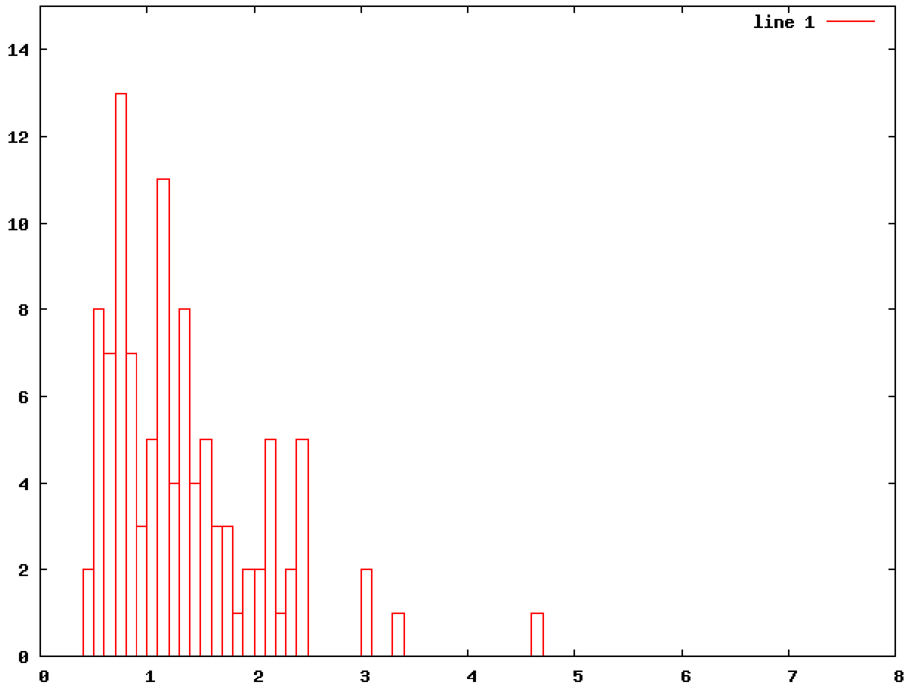

Figure 5: Absolute weekly errors for put testing under H

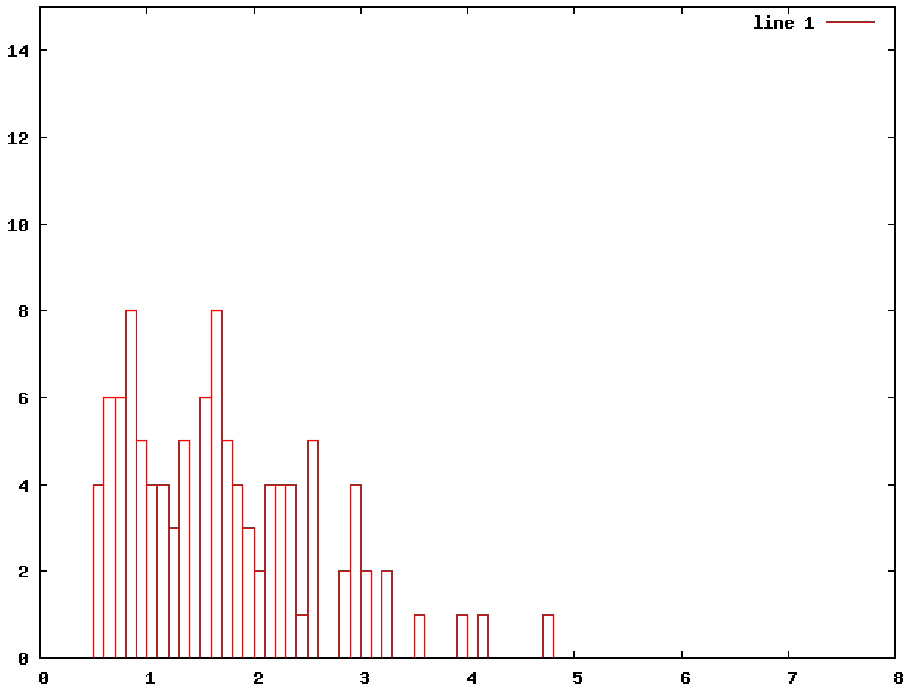

Figure 6: Absolute weekly errors for put testing under SVJ

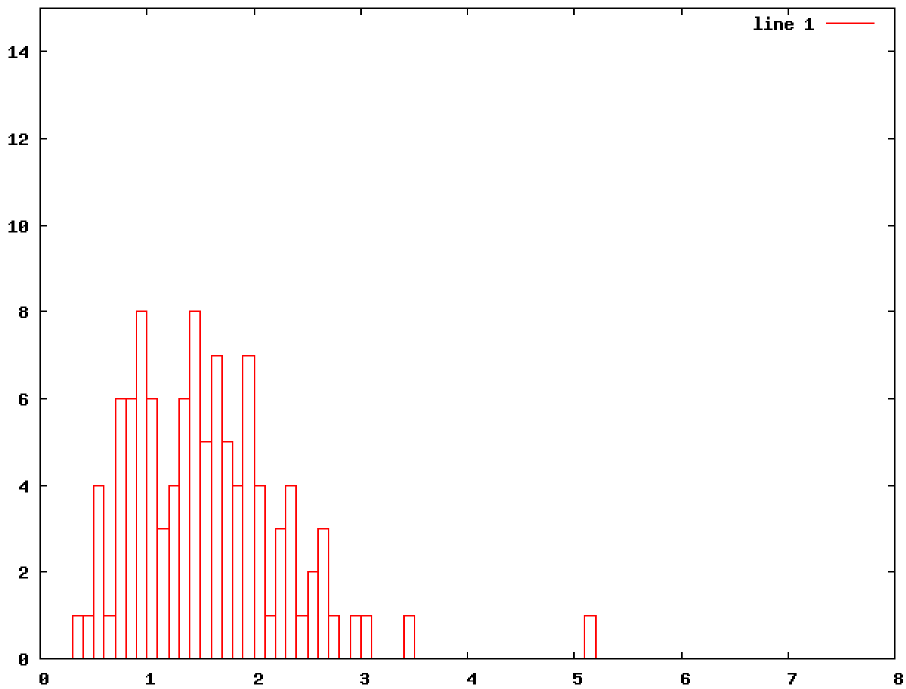

Figure 7: Absolute weekly errors for put testing under VGSA

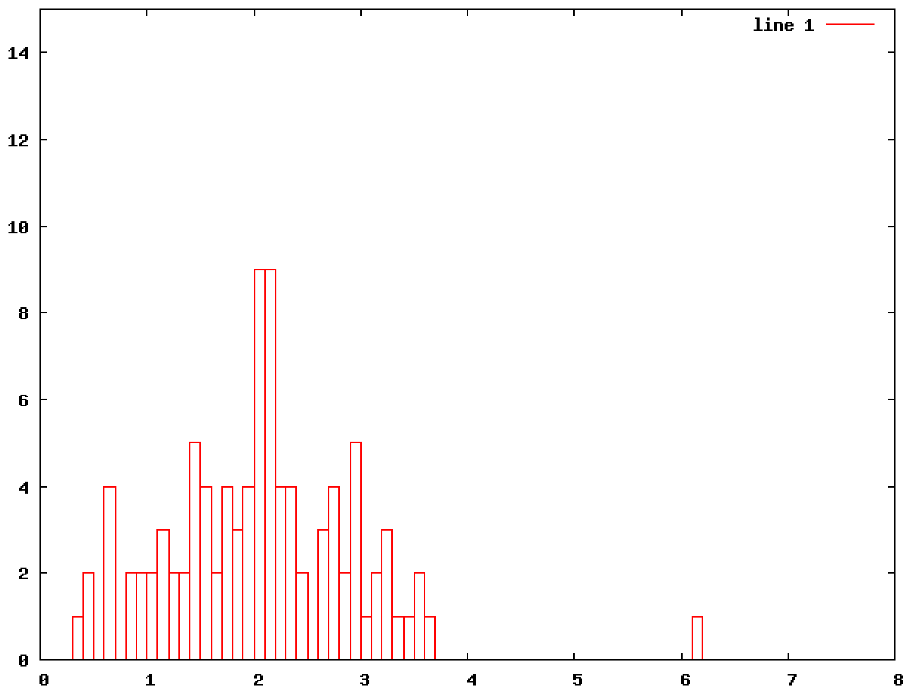

Figure 8: absolute weekly errors for put testing under CGMYSA

long term ($T > 90$ days) deep in the money ($S/K < 0.95$) puts are 9.7305 and 12.66%, respectively; however, they are 1.8145 and 38.47% for long term ($T > 90$) out of the money ($S/K > 1.085$) puts. In general, all the models perform poorly for out of the money puts. Yet, far out of the money options represent about one third of all observations, and they explain a good part of the average weekly errors we discussed in the previous section. As for calls, for deep in the money puts($S/K < 0.95$), both absolute and absolute percentage errors increase quickly with time to maturity, from about 2% for short term options($T < 30$) to about 15% for long term options($T > 90$). For puts that are close to at the money($S/K \in [0.95, 1.05]$), absolute errors increase with time to maturity, but absolute percentage errors for different maturities do not show any regular trend, and they appear model dependent. Overall, CGMY has the best deep in the money and far out of the money performance, while BS has the best performance around at the money.

Second, if we compare the performance of the stochastic volatility models with their constant volatility counterparts, we observe that errors for the constant volatility models are consistently smaller, in almost all the moneyness/maturity categories. This is in contrast with the results of the out of sample pricing test on the call. This might be due to the fact that option prices show high autocorrelation, and models with more parameters will perform better as a result of overfitting.

Table 17: Put Errors According to Maturity and Moneyness for BS

Models	Moneyness	Call Options Days to Expiration		
	(S/K)	($\leq$ 30 days)	(30 $\sim$ 90 days)	($>$ 90 days)
BS	$X < 0.95$	0.9799 (0.7955)	4.2881 (1.8935)	10.4951 (4.0679)
		0.0276 (0.0251)	0.0696 (0.0343)	0.1431 (0.0620)
	$0.95 \sim 0.985$	0.8857 (0.6948)	2.4335 (1.5066)	6.3939 (3.0279)
		0.0537 (0.0424)	0.0998 (0.0581)	0.1760 (0.0708)
	$0.985 \sim 1.015$	0.6882 (0.5359)	1.4793 (1.0358)	4.8883 (2.6060)
		0.1364 (0.1462)	0.1047 (0.0679)	0.1838 (0.0843)
	$1.015 \sim 1.05$	0.6165 (0.5699)	0.8642 (0.8255)	2.6884 (2.6978)
		0.2303 (0.2042)	0.0902 (0.0698)	0.1312 (0.1155)
	$1.05 \sim 1.085$	0.9815 (0.7104)	1.4339 (0.8755)	1.8029 (1.5608)
		0.4878 (0.2348)	0.2728 (0.1570)	0.1106 (0.0839)
	$X > 1.085$	1.0956 (0.7353)	1.7079 (0.8750)	2.2343 (1.0672)
		0.7646 (0.1904)	0.6330 (0.2440)	0.5324 (0.3028)

Table 18: Put Errors According to Maturity and Moneyness for MJD

Models	Moneyness	Call Options Days to Expiration		
	(S/K)	($\leq$ 30 days)	(30 $\sim$ 90 days)	($>$ 90 days)
MJD	$X < 0.95$	0.9953 (0.7963)	4.7130 (2.2216)	11.2194 (4.3458)
		0.0280 (0.0247)	0.0756 (0.0399)	0.1560 (0.0672)
	$0.95 \sim 0.985$	0.9186 (0.7617)	2.7915 (1.6708)	8.1074 (2.7837)
		0.0560 (0.0470)	0.1183 (0.0742)	0.2263 (0.0721)
	$0.985 \sim 1.015$	0.9591 (0.7456)	2.5145 (1.3995)	7.1554 (3.0079)
		0.2009 (0.2374)	0.1906 (0.1157)	0.2814 (0.1338)
	$1.015 \sim 1.05$	1.0010 (0.7864)	1.8946 (1.3463)	5.4527 (3.1061)
		0.4401 (0.5048)	0.2444 (0.2092)	0.3243 (0.2504)
	$1.05 \sim 1.085$	1.0050 (0.8542)	1.4640 (1.2469)	5.7097 (3.6280)
		0.5678 (0.6063)	0.2825 (0.3070)	0.3676 (0.2101)
	$X > 1.085$	1.0687 (0.7979)	1.5194 (1.1683)	2.2553 (2.1201)
		0.8986 (1.0534)	0.5577 (0.4391)	0.4571 (0.3710)

Table 19: Put Errors According to Maturity and Moneyness for VG

Models	Moneyness (S/K)	Call Options Days to Expiration		
		($\leq$ 30 days)	(30 $\sim$ 90 days)	(> 90 days)
VG	$X < 0.95$	0.9766 (0.7756)	4.1838 (1.8927)	11.2894 (4.0408)
		0.0276 (0.0244)	0.0677 (0.0324)	0.1576 (0.0666)
	0.95 $\sim$ 0.985	0.9494 (0.7911)	2.9095 (1.5883)	8.3903 (2.5064)
		0.0575 (0.0481)	0.1234 (0.0732)	0.2344 (0.0657)
	0.985 $\sim$ 1.015	1.0155 (0.7837)	2.6879 (1.3351)	7.1590 (2.6122)
		0.2120 (0.2488)	0.2042 (0.1132)	0.2764 (0.1003)
	1.015 $\sim$ 1.05	0.9561 (0.7374)	1.8843 (1.1872)	5.1506 (2.1119)
		0.4188 (0.4571)	0.2446 (0.1864)	0.2868 (0.1083)
	1.05 $\sim$ 1.085	0.7772 (0.6920)	1.1633 (0.8610)	4.7186 (3.3069)
		0.4167 (0.3852)	0.2414 (0.2208)	0.2907 (0.1534)
	$X > 1.085$	0.7544 (0.6639)	1.0924 (0.8249)	1.7524 (1.7485)
		0.5205 (0.2995)	0.4047 (0.2850)	0.3357 (0.2239)

Table 20: Put Errors According to Maturity and Moneyness for CGMY

Models	Moneyness (S/K)	Call Options Days to Expiration		
		($\leq$ 30 days)	(30 $\sim$ 90 days)	(> 90 days)
CGMY	$X < 0.95$	0.9948 (0.7740)	4.0120 (1.8251)	9.7305 (5.4108)
		0.0279 (0.0244)	0.0640 (0.0307)	0.1266 (0.0657)
	0.95 $\sim$ 0.985	0.9169 (0.7810)	2.3891 (1.5713)	7.1809 (2.8060)
		0.0550 (0.0453)	0.1000 (0.0689)	0.2004 (0.0763)
	0.985 $\sim$ 1.015	0.7940 (0.6961)	2.0355 (1.2916)	5.3461 (2.8139)
		0.1691 (0.2350)	0.1530 (0.1071)	0.2016 (0.0999)
	1.015 $\sim$ 1.05	0.7395 (0.7162)	1.4173 (1.1113)	3.7898 (2.3182)
		0.3438 (0.4604)	0.1761 (0.1686)	0.2096 (0.1208)
	1.05 $\sim$ 1.085	0.6908 (0.7810)	1.0162 (0.8116)	3.8005 (2.4446)
		0.3527 (0.3813)	0.2040 (0.1993)	0.2354 (0.1218)
	$X > 1.085$	0.7523 (0.7824)	1.1709 (0.9288)	1.8145 (1.3263)
		0.4794 (0.3112)	0.4051 (0.2710)	0.3847 (0.2468)

Table 21: Put Errors According to Maturity and Moneyness for H

Models	Moneyness	Call Options Days to Expiration		
	(S/K)	$(\leq 30$ days)	$(30 \sim 90$ days)	$(> 90$ days)
Heston	$X < 0.95$	1.0132 (0.7846)	3.5375 (1.7915)	10.7414 (4.2063)
		0.0286 (0.0250)	0.0570 (0.0325)	0.1495 (0.0654)
	$0.95 \sim 0.985$	1.0339 (0.8521)	2.8084 (1.6098)	8.2795 (2.5992)
		0.0627 (0.0518)	0.1209 (0.0772)	0.2320 (0.0721)
	$0.985 \sim 1.015$	1.0473 (0.8256)	2.8832 (1.4955)	7.5096 (3.2870)
		0.1791 (0.1445)	0.2151 (0.1170)	0.2935 (0.1290)
	$1.015 \sim 1.05$	0.9733 (0.7808)	2.1698 (1.3300)	5.6259 (3.0900)
		0.3743 (0.2849)	0.2672 (0.1950)	0.3166 (0.1742)
	$1.05 \sim 1.085$	0.8872 (0.8168)	1.6747 (1.2114)	6.4827 (3.9614)
		0.4547 (0.3474)	0.3270 (0.2783)	0.4065 (0.1983)
	$X > 1.085$	0.7411 (0.7300)	1.2732 (1.0301)	2.6539 (2.9174)
		0.5408 (0.4482)	0.4885 (0.3862)	0.4537 (0.3795)

Next, we try to get a sense for whether the models overprice or underprice the options. We focus on BS and CGMY which, as we argued before, show the best overall performance. We compute their percentage errors (as opposed to absolute percentage error) and we report them in tables 25 and 26. A positive (negative) error means the model undervalues (overvalues) the puts. Both models systematically overvalue in the money and at the money options. BS undervalues most of the out of the money options, while CGMY overvalues moderately-out of the money puts($S/K < 1.085$), but also undervalues far out of the money money puts. CGMY actually performs much better for far out of the money options than BS. The percentage error for CGMY is

Table 22: Put Errors According to Maturity and Moneyness for SVJ

Models	Moneyness	Call Options Days to Expiration		
	(S/K)	($\leq$ 30 days)	(30 $\sim$ 90 days)	(> 90 days)
SVJ	$X < 0.95$	0.9859 (0.7705)	3.5358 (1.7421)	11.0605 (3.9346)
		0.0278 (0.0248)	0.0576 (0.0336)	0.1579 (0.0691)
	0.95 $\sim$ 0.985	1.0013 (0.8109)	3.0655 (1.6452)	9.0633 (2.9507)
		0.0610 (0.0499)	0.1294 (0.0722)	0.2539 (0.0814)
	0.985 $\sim$ 1.015	1.0835 (0.7888)	3.1156 (1.6063)	8.5759 (2.9113)
		0.1990 (0.1687)	0.2271 (0.1197)	0.3335 (0.1248)
	1.015 $\sim$ 1.05	1.2384 (0.8241)	2.7819 (1.8212)	7.2019 (3.9606)
		0.5474 (0.5398)	0.3398 (0.2828)	0.4100 (0.2753)
	1.05 $\sim$ 1.085	1.4562 (0.9179)	2.6409 (1.8608)	7.4691 (3.4885)
		0.9910 (0.9941)	0.5300 (0.4813)	0.4923 (0.2360)
	$X > 1.085$	1.2671 (0.9159)	1.9412 (1.7572)	3.9536 (3.7286)
		1.3558 (1.6963)	0.8099 (1.0035)	0.8153 (1.1164)

Table 23: Put Errors According to Maturity and Moneyness for VGSA

Models	Moneyness	Call Options Days to Expiration		
	(S/K)	($\leq$ 30 days)	(30 $\sim$ 90 days)	(> 90 days)
VGSA	$X < 0.95$	1.3162 (1.0248)	4.5451 (1.9614)	11.7209 (4.1454)
		0.0358 (0.0297)	0.0761 (0.0430)	0.1686 (0.0795)
	0.95 $\sim$ 0.985	1.2299 (0.9768)	3.5944 (1.7490)	9.4924 (2.6250)
		0.0741 (0.0596)	0.1519 (0.0801)	0.2662 (0.0757)
	0.985 $\sim$ 1.015	1.2764 (0.8868)	3.5609 (1.5770)	8.8599 (4.0169)
		0.2524 (0.2892)	0.2637 (0.1289)	0.3483 (0.1664)
	1.015 $\sim$ 1.05	1.2189 (0.8269)	2.9271 (1.5536)	7.4484 (3.8255)
		0.5177 (0.5775)	0.3607 (0.2394)	0.4397 (0.2898)
	1.05 $\sim$ 1.085	0.9971 (0.7461)	2.1659 (1.4011)	7.3504 (4.3733)
		0.6071 (0.6670)	0.4234 (0.3267)	0.4778 (0.2476)
	$X > 1.085$	0.6412 (0.5765)	1.1181 (0.9701)	2.9862 (3.1409)
		0.5366 (0.6047)	0.4237 (0.3695)	0.5482 (0.5118)

Table 24: Errors According to Maturity and Moneyness for CGMYSA

Models	Moneyness	Call Options Days to Expiration		
	(S/K)	($\leq$ 30 days)	(30 $\sim$ 90 days)	($>$ 90 days)
CGMYSA	$X < 0.95$	1.1368 (0.8550)	4.3278 (2.0152)	11.3931 (3.4941)
		0.0309 (0.0256)	0.0708 (0.0407)	0.1604 (0.0700)
	0.95 $\sim$ 0.985	1.1379 (0.8742)	3.4107 (1.5750)	9.5421 (2.5061)
		0.0682 (0.0540)	0.1420 (0.0681)	0.2715 (0.0750)
	0.985 $\sim$ 1.015	1.2819 (0.8772)	3.6317 (1.5283)	8.7596 (3.2343)
		0.2392 (0.2754)	0.2683 (0.1222)	0.3513 (0.1513)
	1.015 $\sim$ 1.05	1.5484 (0.9354)	3.5155 (1.6549)	8.2213 (3.6765)
		0.6210 (0.6407)	0.4287 (0.2685)	0.4828 (0.2871)
	1.05 $\sim$ 1.085	1.8234 (0.9615)	3.2946 (1.7790)	8.7782 (4.3432)
		1.1866 (1.1841)	0.6307 (0.4348)	0.5706 (0.2471)
	$X > 1.085$	1.7369 (0.9906)	2.6672 (1.6179)	5.0956 (3.4443)
		1.8217 (1.8214)	1.1625 (1.1635)	1.0699 (0.7509)

only half of BS in this moneyness category. For example, the percentage error of BS is 76% and 63% for short term and medium term puts, respectively, but only 35% and 30% for CGMY. We also observe that percentage errors for far out of the money options are substantially larger for both models.

While BS has smaller absolute percentage errors for close to at the money options, the size of the percentage error for CGMY is smaller in almost all the moneyness-maturity categories. This means that CGMY actually has smaller systematic bias than BS. If we exclude the far out of the money options, we found much smaller moneyness bias for CGMY. For example, the percentage errors for short term options for BS range from -7% to 48%, while they range from -17% to 0 for CGMY. This comparison is even more striking for medium term options: They range from -9% to 22% for BS, but only from -6% to -11% for CGMY. The performance of CGMY is more consistent across different degrees of moneyness than BS.

Table 25: Put Errors According to Maturity and Moneyness for BS

Models	Moneyness (S/K)	Call Options Days to Expiration		
		($\leq$ 30 days)	(30 $\sim$ 90 days)	($>$ 90 days)
BS	$X < 0.95$	-0.0062 (0.0388)	-0.0694 (0.1431)	-0.1431 (0.2929)
	0.95 $\sim$ 0.985	-0.0160 (0.0739)	-0.0953 (0.2015)	-0.1754 (0.3582)
	0.985 $\sim$ 1.015	-0.0702 (0.2340)	-0.0919 (0.2023)	-0.1805 (0.3723)
	1.015 $\sim$ 1.05	0.1810 (0.4394)	0.0131 (0.1163)	-0.1168 (0.2673)
	1.05 $\sim$ 1.085	0.4847 (0.9990)	0.2166 (0.4898)	-0.0268 (0.1463)
	$X > 1.085$	0.7642 (1.5404)	0.6314 (1.2870)	0.5020 (1.0636)

Table 26: Put Errors According to Maturity and Moneyness for CGMY

Models	Moneyness (S/K)	Call Options Days to Expiration		
		($\leq$ 30 days)	(30 $\sim$ 90 days)	($>$ 90 days)
CGMY	$X < 0.95$	-0.0050 (0.0381)	-0.0614 (0.1279)	-0.0991 (0.2232)
	0.95 $\sim$ 0.985	0.0035 (0.0715)	-0.0779 (0.1815)	-0.1684 (0.3620)
	0.985 $\sim$ 1.015	-0.0806 (0.3214)	-0.1088 (0.2653)	-0.1257 (0.3131)
	1.015 $\sim$ 1.05	-0.1755 (0.6500)	-0.1067 (0.3059)	-0.1138 (0.3120)
	1.05 $\sim$ 1.085	-0.0067 (0.5195)	-0.0744 (0.3130)	0.0080 (0.2654)
	$X > 1.085$	0.3544 (0.8387)	0.2998 (0.7122)	0.3301 (0.7320)

6 Conclusions

There are a number of extensions of the Black and Scholes (1973) model that appear to exhibit good empirical performance. In this paper we focus on eight models, all of them belonging to the class of Lévy models. However, previous test are mostly restricted to European calls. In this paper we consider their performance for American options and, in particular, their put-call robustness. To do that, we first need to extend the approximation method of Barone-Adesi and Whaley (1987) to the infinite activity Lévy models. We test our extension numerically and show that it is an efficient and accurate method to calculate the early exercise premium of the American options.

We find that, although all of the newly developed jump/stochastic volatility models improve the performance of the Black and Scholes (1973) model on the call side, their put side performance is worse than that of the Black and Scholes (1973) model, with the only exception (possibly) of the Carr, Geman, Madan, and Yor (2002) model. The sharp difference in performance between the call side and the put side seems to indicate that the overfitting effect of the multi-parameter models is non-negligible. In addition, the superior performance of the Carr, Geman, Madan, and Yor (2002) model shows that the log return of the underlying is closest to the tempered stable process, as opposed to the models with stochastic volatility.

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